SAMPLE PAPERS

CBSE EXAM 2024

Class 12th

Sub : Mathematics

Marking Scheme links for all papers is given at the end of these papers.

20 Sets

Sample Paper 1

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $f(x)=\begin{cases} ax+3, & x \leq 2 \\ a^2x-1, & x>2 \end{cases}$, then the values of a for which f is continuous for all x are

(a) 1 and -2

(b) 1 and 2

(c) -1 and 2

(d) -1 and -2

2. The function $f(x)=x^2$, for all real x, is

(a) decreasing

(b) increasing

(c) neither decreasing nor increasing

(d) none of the above

3. The function $f(x)=x^3$ has a

(a) local minima at $x=0$

(b) local maxima at $x=0$

(c) point of inflexion at $x=0$

(d) none of the above

4. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx$ is equal to

(a) $\frac{1}{2}\sqrt{1+x}+C$

(b) $\frac{2}{3}(1+x)^{3/2}+C$

(c) $\sqrt{1+x}+C$

(d) $2(1+x)^{3/2}+C$

5. $\int_0^{\pi/2} \frac{\sin x-\cos x}{1+\sin x\cos x}dx$ is equal to

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π

6. The order of the differential equation of all circles of radius a is

(a) 2 (b) 3

(c) 4 (d) 1

7. The derivative of $\log|x|$ is

(a) $\frac{1}{x}$, $x > 0$ (b) $\frac{1}{|x|}$, $x \neq 0$

(c) $\frac{1}{x}$, $x \neq 0$ (d) None of these

8. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ are two sets and function $f : A \to B$ is defined by $f(x) = x + 2$, $\forall x \in A$, then the function f is

(a) bijective (b) onto

(c) one-one (d) many-one

9. $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$ is equal to

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

10. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$ and $adj\,(A) + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the values of x and y are

(a) 1, 1 (b) ± 1, 1

(c) 1, 0 (d) None of these

11. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$ (b) $\log 64$

(c) $\log 81$ (d) $\log 27$

12. Solution of $(x + 2y^3)\,dy = y\,dx$ is

(a) $x = y^3 + cy$ (b) $x + y^3 = cy$

(c) $y^2 - x = cy$ (d) none of these

13. Integrating factor of differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$

(a) $\cos x$ (b) $\tan x$

(c) $\sec x$ (d) $\sin x$

14. Two vectors $\vec{a}$ and $\vec{b}$ are parallel and have same magnitude, then

(a) they have the same direction
(b) they are equal
(c) they are not equal
(d) they may or may not be equal

15. If the position vectors of the vertices A, B, C of a triangle ABC are $7\hat{j}+10\hat{k}, -\hat{i}+6\hat{j}+6\hat{k}$ and $-4\hat{i}+9\hat{j}+6\hat{k}$ respectively, then triangle is

(a) equilateral
(b) isosceles
(c) scalene
(d) right angled and isosceles also

16. If α, β, γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is equal to

(a) 2
(b) 1
(c) 0
(d) -1

17. If $P(A \cup B) = 0.83$, $P(A) = 0.3$ and $P(B) = 0.6$, then the events will be

(a) dependent
(b) independent
(c) cannot say anything
(d) None of the above

18. Two dice are thrown n times in succession. The probability of obtaining a doublet six atleast once is

(a) $\left(\frac{1}{36}\right)^n$
(b) $1-\left(\frac{35}{36}\right)^n$
(c) $\left(\frac{1}{12}\right)^n$
(d) None of these

19. **Assertion:** Let $A = \{-1,1,2,3\}$ and $B = \{1,4,9\}$ where $f: A \to B$ given by $f(x) = x^2$, then f is a many-one function.
Reason: If $x_1 \neq x^2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_{1,} x_2 \in$ domain then f is one-one or else many

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

20. **Assertion:** If $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$, then $(A^T)A = I$

Reason: For any square matrix A, $(A^T)^T = A$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b): 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.

22. Find $\int \frac{\sin^6 x}{\cos^8 x} dx$.

OR

Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$.

23. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

OR

Find the unit vector in the direction of the sum of vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

24. Find the vector equation of the line which passes through the point $(3, 4, 5)$ and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

25. Prove that if E and F are independent events, then the events E and F' are also independent.

Section - C

This section comprises of short answer-type questions (LA) of 3 marks each.

26. Write the value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

27. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

28. Find the value of k, so that the following functions is continuous at $x = 2$. $f(x) = \begin{cases} \frac{x^3 - x^2 - 16x + 20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$

OR

Find $\frac{dy}{dx}$ at $x = 1,\ y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

29. The volume of a sphere is increasing at the rate of 8 cm³/s. Find the rate of which its surface area is increasing when the radius of the sphere is 12 cm.

OR

Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

30. Evaluate $\int \frac{2\cos x}{3\sin^2 x} dx$.

31. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Show that the line lines

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) \text{ and}$$
$$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$ intersect. Also, find their point of intersection.

33. Find the value of $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$.

OR

Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$.

34. Show that the differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$ is homogeneous.

Find the particular solution of this differential equation, given that $y = \frac{\pi}{4}$, when $x = 1$.

35. Maximise $Z = 5x + 3y$ subject to the constraints: $3x + 5y \le 15$; $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$.

OR

Minimize $Z = 3x + 5y$ such that $x + 3y \ge 3$, $x + y \ge 2$, x, $y \ge 0$.

Section - E

Case study based questions are compulsory.

36. A market analysis is a quantitative and qualitative assessment of a market. It looks into the size of the market both in volume and in value, the various customer segments and buying patterns, the competition, and the economic environment in terms of barriers to entry and regulation.

Based on the past marketing trends and his own experience, marketing expert suggested to the concerned the segments of market for their products as follows:
The first segment consisted of lower income class, the second segment that of middle income and the third segment

that of high income. The data based on the income of the consumers was readily available. During a particular month in particular year, the agent reported that for three products of the company the following were the sales: There were 200 customers who bought all the three products, 240 customers who bought I and III, 60 customers only products II and II and 80 customers only products only III regardless of the market segmentation groups. Based on the market segmentation analysis, for product I, the percentage for the income groups are given as (40%, 20% and 40%), for product II (30%, 20% and 50%), for product III (10%, 50% and 40%).

(i) Taking the suitable variable form the system of equation that represent given problem.

(ii) Using matrix method, find out the number of persons in the lower income, middle income and higher income class in the region referred.

37. In apparels industries retailers have an interesting conundrum facing them. On one hand, consumers are more drawn to hot promotional deals than ever before. The result of this is that they sell more units (of product) for less money, and this adversely impacts comp store sales.

Arvind Fashions knows that the it can sell 1000 shirts when the price is ₹ 400 per shirt and it can sell 1500 shirts when the price is ₹ 200 a shirt. Determine

(i) the price function

(ii) the revenue function

(iii) the marginal revenue function.

38. The U.S. Constitution directs the government to conduct a census of the population every 10 years. Population totals are used to allocate congressional seats, electoral votes, and funding for many government programs. The U.S. Census Bureau also compiles information related to income and poverty, living arrangements for children, and marital status. The following joint probability table lists the probabilities corresponding to marital status and sex of persons 18 years and over.14

Sex	Marital Status			
	(R)	(N)	(W)	(D)
(M)	0.282	0.147	0.013	0.043
(F)	0.284	0.121	0.050	0.060

$R \Rightarrow$ Married
$N \Rightarrow$ Never married
$W \Rightarrow$ Widowed
$D \Rightarrow$ Discovered or separated
$M \Rightarrow$ Male
$F \Rightarrow$ Male

Suppose a U.S. resident 18 years or older is selected at random.

(i) Find the probability that the person is female and widowed.

(ii) Suppose the person is male. What is the probability that he was never married?

(iii) Suppose the person is married. What is the probability that the person is female?

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Sample Paper 2

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
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5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ holds good for all

(a) $|x| \leq 1$ (b) $1 \geq x \geq 0$

(c) $|x| \leq \frac{1}{\sqrt{2}}$ (d) none of these

2. If A and B are two symmetric matrices of same order. Then, the matrix $AB - BA$ is equal to

(a) a symmetric matrix (b) a skew-symmetric matrix

(c) a null matrix (d) the identity matrix

3. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) -1

4. If $x = e^{y+e^{y+e^{y+\dots}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x}$ (b) $\frac{1-x}{x}$

(c) $\frac{x}{1+x}$ (d) None of these

5. Let $f(x) = \tan x - 4x$, then in the interval $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$, $f(x)$ is

(a) a decreasing function (b) an increasing function

(c) a constant function (d) none of these

6. Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

7. The value of $\int_0^1 \frac{dx}{e^x + e}$ is

(a) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$ (b) $\log\left(\frac{1+e}{2}\right)$

(c) $\frac{1}{e}\log(1+e)$ (d) $\log\left(\frac{2}{1+e}\right)$

8. If $f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \neq 0$, then $f(x)$ is equal to

(a) $x^2 - 2$ (b) $x + 2$

(c) $x^2 + 2$ (d) $2x^2 - 5$

9. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ are two sets and function $f : A \to B$ is defined by $f(x) = x + 2$, $\forall x \in A$, then the function f is

(a) bijective (b) onto

(c) one-one (d) many-one

10. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then $A \cdot (adj\ A)$ is equal to

(a) A (b) $|A|$

(c) $|A| \cdot I$ (d) None of these

11. The area of enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(a) 12 sq units (b) 13 sq units

(c) $13\frac{1}{2}$ sq units (d) 14 sq units

12. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

(a) 1 (b) 2

(c) 3 (d) 4

13. The general solution of the differential equation $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$ is

(a) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$ (b) $\log\tan\left(\frac{y}{4}\right) = c - 2\sin\frac{x}{2}$

(c) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = c - 2\sin x$ (d) $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

14. Solution of the equation $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is

(a) $\sin\frac{x}{y} = cx$

(b) $\sin\frac{y}{x} = cx$

(c) $\sin\frac{x}{y} = cy$

(d) $\sin\frac{y}{x} = cy$

15. The foot of the perpendicular from (0, 2, 3) to the line $\frac{x+3}{5} = \frac{y=1}{2} = \frac{z+4}{3}$ is

(a) $(-2, 3, 4)$

(b) $(2, -1, 3)$

(c) $(2, 3, -1)$

(d) $(3, 2, -1)$

16. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is

(a) $\vec{a}$

(b) $\vec{b}$

(c) $\vec{a} + \vec{b}$

(d) $\vec{a} - \vec{b}$

17. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$

(b) $\frac{1}{6}$

(c) $\frac{5}{6}$

(d) $\frac{1}{3}$

18. Objective function of a linear programming problem is

(a) a constraint

(b) a function to be optimized

(c) a relation between the variables

(d) none of the above

19. **Assertion:** $\int_0^{2\pi} \sin^3 x\, dx = 0$

Reason: $\sin^3 x$ is an odd function.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** The pair of lines given by $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + k)$ and $\vec{r} = 2\hat{i} - k + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect.

Reason: Two lines intersect each other, if they are not parallel and shortest distance $= 0$.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b): 2 \text{ divides } (a - b)\}$. Write the equivalence class [0].

22. Evaluate $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$.

OR

Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$.

23. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

OR

Find the unit vector in the direction of the sum of vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

24. The x-coordinate of point on the line joining the points $P(2,2,1)$ and $Q(5,1,-2)$ is 4. Find its z-coordinate.

25. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then find $P(A \cup B)$

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Write the value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

27. If $\begin{vmatrix} 3x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.

28. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

29. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

OR

Find the intervals in which the function

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51 \text{ is}$$

(i) strictly increasing

(ii) strictly decreasing.

30. Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

OR

Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.

31. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the values of p, so that the lines

$$l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$$

and $$l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

33. Evaluate $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$.

34. Solve the differential equation $x \log|x| \frac{dy}{dx} + y = \frac{2}{x} \log|x|$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that $y = 1$, when $x = 0$.

35. Maximize $Z = -x + 2y$, subject to the constraints :

$$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$$

OR

Maximize $Z = x + y$, subject to $x - y \leq -1,\ x + y \leq 0,\ x,\ y \geq 0$.

Section - E

Case study based questions are compulsory.

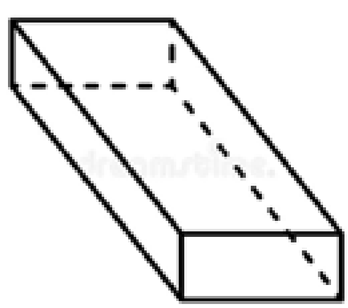
36. Rajneesh do outsourcing work for companies and runs a form processing agency. He collect form from different office and then extract data and record data on computer. In his office three employees Vikas, Sarita and Ishaan process incoming copies of a form. Vikas process 50% of the forms. Sarita processes 20% and Ishaan the remaining 30% of the forms. Vikas has an error rate of 0.06, Sarita has an error rate of 0.04 and Ishaan has an error rate of 0.03.

Based on the above information answer the following questions.

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vikas.

37. Parallelepiped is the Greek word, which essentially means the object that has a parallel plane. Principally, the Parallelepiped is framed by the six parallelogram sides which bring about the prism or the 3D figure, and it consists of the parallelogram base. It can be categorized as anything but the polyhedron, where 3 sets of the parallel faces are made to combine for framing a three-dimensional (3D) shape that has six faces. The cube, cuboid, and rhomboid are the three exceptional cases. The Rectangular Parallelepiped consists of six faces in a rectangular shape.



The sum of the surface area of a rectangular parallelopied with sides of x, $2x$ and $\frac{x}{3}$ and a shape of radius of y is given to be constant.

On the basis of above information, answer the following questions.

(i) If S is the constant, then find the relation between S, x and y.

(ii) If the combined volume is denoted by V , then find the relation between V, x and y.
(iii) Find the relation between x and y when the volume V is minimum.

OR

If at $x = 3y$, volume V is minimum, then find the value of minimum volume and the value of S.

38. A manufacturing company has two service departments, S_1, S_2 and four production departments P_1, P_2, P_3 and P_4.
Overhead is allocated to the production departments for inclusion in the stock valuation. The analysis of benefits received by each department during the last quarter and the overhead expense incurred by each department were:

Service Department	**Percentages to be allocated to departments**					
	S_1	S_2	P_1	P_2	P_3	P_4
S_1	0	20	30	25	15	10
S_2	30	0	10	35	20	5
Direct overhead expense ₹ '000	20	40	25	30	20	10

You are required to find out following using matrix method.
(i) Express the total overhead of the service departments in the form of simultaneous equations.
(ii) Express these equations in a matrix form and solve for total overhead of service departments using matrix inverse method.
(iii) Determine the total overhead to be allocated from each of S_1 and S_2 to the production department.

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Sample Paper 3

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
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Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $y = \log[\sin(x^2)]$, $0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ at $x = \frac{\sqrt{\pi}}{2}$ is

(a) 0 (b) 1

(c) $\pi/4$ (d) $\sqrt{\pi}$

2. The function $f(x) = 2 - 3x$ is

(a) decreasing (b) increasing

(c) neither decreasing nor increasing (d) none of the above

3. The radius of a cylinder is increasing at the rate of 3m/s and its altitude is decreasing at the rate of 4m/s . The rate of change of volume when radius is 4m and altitude is 6m, is

(a) $80\pi m^3/s$ (b) $144\pi m^3/s$

(c) $80 m^3/s$ (d) $64 m^3/s$

4. $\int \sqrt{1+\cos x}\, dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right) + C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right) + C$

(c) $2\sqrt{2}\sin\left(\frac{x}{2}\right) + C$ (d) $\frac{1}{2}\sin\left(\frac{x}{2}\right) + C$

5. The value of $\int_{-2}^{2} (x\cos x + \sin x + 1)\, dx$ is

(a) 2 (b) 0

(c) -2 (d) 4

6. The function $f(x) = x^2 + bx + c$, where b and c are real constants, describes

(a) one-one mapping
(b) onto mapping
(c) not one-one but onto mapping
(d) neither one-one nor onto mapping

7. If $|x| \le 1$, which of the following four is different from the other three?

(a) $\sin(\cos^{-1}x)$
(b) $\cos(\sin^{-1}x)$
(c) $\sqrt{1-x^2}$
(d) $\frac{\sqrt{1-x^2}}{x}$

8. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined, then B is of the type

(a) 4×4
(b) 3×4
(c) 4×3
(d) 3×3

9. If A is a matrix of order 3 such that $A(adj\ A) = 10I$, then $|adj\ A|$ is equal to

(a) 1
(b) 10
(c) 100
(d) $10I$

10. If $f(x) = \begin{cases} \frac{3\sin\pi x}{5x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) $\frac{\pi}{10}$
(b) $\frac{3\pi}{10}$
(c) $\frac{3\pi}{2}$
(d) $\frac{3\pi}{5}$

11. The area of the region bounded by the lines $y = mx$, $x = 1$, $x = 2$ and X-axis is 6 sq units, then m is equal to

(a) 3
(b) 1
(c) 2
(d) 4

12. Solution of $\frac{dy}{dx} + y\sec x = \tan x$ is

(a) $y(\sec x + \tan x) = \sec x + \tan x - x + c$
(b) $y = \sec x + \tan x - x + c$
(c) $y(\sec x + \tan x) = \sec x + \tan x + x + c$
(d) none of the above

13. The equation of the curve, whose slope at any point different from origin is $y + \frac{y}{x}$, is

(a) $y = cxe^x, c \neq 0$
(b) $y = xe^x$
(c) $xy = e^x$
(d) $y + xe^x = c$

14. The differential equation representing the family of curve $y^2 = (x + \sqrt{c})$, where c is positive perimeter, is of

(a) order 1, degree 3
(b) order 2, degree 2
(c) degree 3, order 1
(d) degree 4, order 4

15. Let $\vec{a}$ and $\vec{b}$ be two non-parallel unit vectors in a plane. If the vectors $(\alpha\vec{a} + \vec{b})$ bisects the internal angle between $\vec{a}$ and $\vec{b}$, then α is equal to

(a) 1
(b) 1/2
(c) 4
(d) 2

16. If α, β, γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(a) 2
(b) 1
(c) 0
(d) -1

17. Which of the following triplets gives the direction cosines of a line?

(a) $< 1, 1, 1 >$
(b) $< 1, -1, 1 >$
(c) $< 1, -1, -1 >$
(d) $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$

18. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

(a) $\frac{1}{72}$
(b) $\frac{3}{73}$
(c) $\frac{5}{72}$
(d) $\frac{1}{74}$

19. **Assertion:** Two dice are tossed the following two events A and B are

$A = \{(x, y): x + y = 11\}$,

$B = \{(x, y): x \neq 5\}$ independent events.

Reason: E_1 and E_2 are independent events, then

$$P(E_1 \cap E_2) = P(E_1)P(E_2)$$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

20. **Assertion:** If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, then value of $(a-b-c)$ is 1.

Reason: A square matrix $A=[a_{ij}]$ is said to be skew-symmetric if $A'=-A$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $R=\{(a,a^3)$: a is a prime number less than $5\}$ be a relation. Find the range of R.

22. Find $\int \frac{\sin^2 x-\cos^2 x}{\sin x\cos x}dx$.

OR

Find $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$.

23. Find the position vector of a point which divides the join of points with position vectors $\vec{a}-2\vec{b}$ and $2\vec{a}+\vec{b}$ externally in the ratio $2:1$.

OR

If $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$, then find a unit vector parallel to the vector $\vec{a}+\vec{b}$.

24. If a line makes angles 90°, 135°, 45° with then x, y and z axes respectively, find the direction consines.

25. If $P(not\ A)=0.7$, $P(B)=0.7$ and $P(B/A)=0.5$, then find $P(A/B)$.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

27. Find $|AB|$, if $A=\begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

28. Determine the value of k for which the following function is continuous at $x=3$:

$$f(x)=\begin{cases} \frac{(x+3)^2-36}{x-3}, & x\neq 3 \\ k, & x=3 \end{cases}$$

OR

Determine the value of constant k so that the function $f(x) = \begin{cases} \frac{kx}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

29. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

OR

The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue we mean the rate of change of total revenue with respect to the number of times sold at an instant.

30. Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$.

31. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a} + \hat{b} + \hat{c}|$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Prove that the line through $A(0, -1, -1)$ and $B(4,5,1)$ intersects the line through $C(3,9,4)$ and $D(-4,4,4)$.

33. Find : $\int \frac{3x + 5}{x^2 + 3x - 18}\,dx$.

OR

Prove that $\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$, hence evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\,dx$.

34. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that $y = 1$, when $x = 0$.

35. Maximize $Z = 3x + 4y$, subject to the constraints; $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

OR

Minimize $Z = -3x + 4y$ subject to the constraints

$$\begin{aligned} x + 2y &\leq 8, \\ 3x + 2y &\leq 12, \\ x &\geq 0,\ y \geq 0 \end{aligned}$$

Section - E

Case study based questions are compulsory.

36. A car carrier trailer, also known as a car-carrying trailer, car hauler, or auto transport trailer, is a type of trailer or semi-trailer designed to efficiently transport passenger vehicles via truck. Commercial-size car carrying trailers are commonly used to ship new cars from the manufacturer to auto dealerships. Modern car carrier trailers can be open or enclosed. Most commercial trailers have built-in ramps for loading and off-loading cars, as well as power hydraulics to raise and lower ramps for stand-alone accessibility.

A transport company uses three types of trucks T_1, T_2 and T_3 to transport three types of vehicles V_1, V_2 and V_3. The capacity of each truck in terms of three types of vehicles is given below:

	V_1	V_2	V_3
T_1	1	3	2
T_2	2	2	3
T_3	3	2	2

Using matrix method find:

(i) The number of trucks of each type required to transport 85, 105 and 110 vehicles of V_1, V_2 and V_3 types respectively.

(ii) Find the number of vehicles of each type which can be transported if company has 10, 20 and 30 trucks of each type respectively.

37. Hindustan Pencils Pvt. Ltd. is an Indian manufacturer of pencils, writing materials and other stationery items, established in 1958 in Bombay. The company makes writing implements under the brands Nataraj and Apsara, and claims to be the largest pencil manufacturer in India.

HINDUSTAN PENCILS PVT. LTD.

apsara

NATARAJ

Hindustan Pencils manufactures x units of pencil in a given time, if the cost of raw material is square of the pencils produced, cost of transportation is twice the number of pencils produced and the property tax costs ₹ 5000. Then,

(i) Find the cost function $C(x)$.
(ii) Find the cost of producing 21st pencil.
(iii) The marginal cost of producing 50 pencils.

38. OYO Rooms, also known as OYO Hotels & Homes, is an Indian multinational hospitality chain of leased and franchised hotels, homes and living spaces. Founded in 2012 by Ritesh Agarwal, OYO initially consisted mainly of budget hotels.

Data analyst at OYO say that during frequent trips to a certain city, a traveling salesperson stays at hotel A 50% of the time, at hotel B 30% of the time, and at hotel C 20% of the time. When checking in, there is some problem with the reservation 3% of the time at hotel A, 6% of the time at hotel B, and 10% of the time at hotel C. Suppose the salesperson travels to this city.
(i) Find the probability that the salesperson stays at hotel A and has a problem with the reservation.
(ii) Find the probability that the salesperson has a problem with the reservation.
(iii) Suppose the salesperson has a problem with the reservation; what is the probability that the salesperson is staying at hotel A?

□□□□□□□

Sample Paper 4

Mathematics (Code-041)

Class XII Session 2022-23

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. The derivative of $\cos^3 x$ with respect to $\sin^3 x$ is

(a) $-\cot x$ (b) $\cot x$

(c) $\tan x$ (d) $-\tan x$

2. Let $f(x) = x - \cos x, x \varepsilon R$, then f is

(a) a decreasing function (b) an odd function

(c) an increasing function (d) none of the above

3. The length of the longest interval, in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

4. $3a\int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to

(a) $a - 1 + (a-1)^{-2}$ (b) $a + a^{-2}$

(c) $a - a^2$ (d) $a^2 + \frac{1}{a^2}$

5. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

6. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to

(a) -1 (b) 0

(c) 1 (d) 2

7. The order of the differential equation of all conics whose centre lie at the origin is given by

(a) 2 (b) 3

(c) 4 (d) 5

8. If $f:[0,\frac{\pi}{2}] \to [0,\infty]$ be a function defined by $y = \sin(\frac{\pi}{2})$, then f is

(a) injective (b) surjective

(c) bijective (d) none of these

9. The area of a parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$, is

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$

(c) $5\sqrt{6}$ (d) $10\sqrt{6}$

10. A mapping $f: n \to N$, where N is the set of natural numbers is define as

$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases}$ for $n \in N$. Then, f is

(a) Surjective but not injective (b) Injective but not surjective

(c) Bijective (d) neither injective nor surjective

11. If $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then for all values of θ

(a) $f(\theta) = 0$ (b) $f(\theta) = 1$

(c) $f(\theta) = -1$ (d) None of these

12. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$ (b) $\log 64$

(c) $\log 81$ (d) $\log 27$

13. The number of solutions of $y' = \frac{y+1}{x-1}, y(1) = 2$ is

(a) zero (b) one

(c) two (d) infinite

14. Solution of $(x+2y^3)\,dy = y\,dx$ is

(a) $x = y^3 + cy$

(b) $x + y^3 = cy$

(c) $y^2 - x = cy$

(d) none of these

15. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{p}$ is a non-zero vector such that $p\vec{r} + (\vec{r}\cdot\vec{b})\vec{a} = \vec{c}$, then $\vec{r}$ is equal to

(a) $\frac{\vec{c}}{p} - \frac{(\vec{b}\cdot\vec{c})\vec{a}}{p^2}$

(b) $\frac{\vec{a}}{p} - \frac{(\vec{c}\cdot\vec{a})\vec{b}}{p^2}$

(c) $\frac{\vec{b}}{p} - \frac{(\vec{a}\cdot\vec{b})\vec{c}}{p^2}$

(d) $\frac{\vec{c}}{p^2} - \frac{(\vec{b}\cdot\vec{c})\vec{a}}{p}$

16. The lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{2} = \frac{y+2}{2} = \frac{z-3}{-2}$ are

(a) parallel

(b) intersecting

(c) at right angle

(d) none of these

17. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{4}$

(d) $\frac{2}{3}$

18. Solution set of the inequality $x \geq 0$ is

(a) half plane on the left of y-axis

(b) half plane on the right of y-axis excluding the points of y-axis

(c) half plane on the right of y-axis including the points on y-axis

(d) none of the above

19. **Assertion:** If $x = at^2$ and $y = 2at$, then $\left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{-1}{16a}$

Reason: $\frac{d^2y}{dx^2} = \left(\frac{dy}{dt}\right)^2 \times \left(\frac{dt}{dx}\right)^2$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** $\int \sin 3x \cos 5x \, dx = \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C$

Reason: $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $A = \{1,2,3\}, B = \{4,5,6,7\}$ and $f = \{(1,4),(2,5),(3,6)\}$ is a function from A to B. State whether f is one and one or not.

22. Evaluate $\int (ax-b)^3 \, dx$.

OR

Evaluate $\int \frac{(1+\log x)^2}{x} dx$.

23. Find $|\vec{x}|$, if for a unit vector $\hat{a}$, $(\vec{x}-\vec{a}) \cdot (\vec{x}+\vec{a}) = 15$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\,\vec{b}\,\vec{c}]$.

24. Find the vector equation of the line passing through the point $A(1,2,-1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

25. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then, find $P(B)$?

OR

Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

27. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

28. If $(x-y)\,e^{\frac{x}{x-y}}=a$, prove that $y\dfrac{dy}{dx}+x=2y$.

29. Show that the function $f(x)=x^3-3x^2+3x,\ x\in R$ is increasing on R.

OR

Find the intervals in which the function $f(x)=\dfrac{3}{2}x^4-4x^3-45x^2+51$ is

(i) strictly increasing

(ii) strictly decreasing.

30. Evaluate $\int\dfrac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$.

OR

Evaluate $\int\dfrac{\sin^6x+\cos^6x}{\sin^2x\cos^2x}\,dx$.

31. Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a}=21$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. If $A=\begin{bmatrix}2&3&10\\4&-6&5\\6&9&-20\end{bmatrix}$, find A^{-1}.

Using A^{-1} solve the system of equations

$\dfrac{2}{x}+\dfrac{3}{y}+\dfrac{10}{z}=2,\ \dfrac{4}{x}-\dfrac{6}{y}+\dfrac{5}{z}=5$ and $\dfrac{6}{x}+\dfrac{9}{y}-\dfrac{20}{z}=-4$

33. Show that the lines $\dfrac{x+1}{3}=\dfrac{y+3}{5}=\dfrac{z+5}{7}$ and $\dfrac{x-2}{1}=\dfrac{y-4}{3}=\dfrac{z-6}{5}$ intersect. Also, find their point intersection.

34. Find the particular solution of the differential equation $\log\left(\dfrac{dy}{dx}\right)=3x+4y$, given that $y=0$, when $x=0$.

OR

Find the particular solution of the differential equation $x(1+y^2)\,dx-y(1+x^2)\,dy=0$, given that $y=1$, when $x=0$.

35. Maximize $Z=3x+2y$ subject to $x+2y\le 0$, $3x+y\le 15$, x, $y\ge 0$.

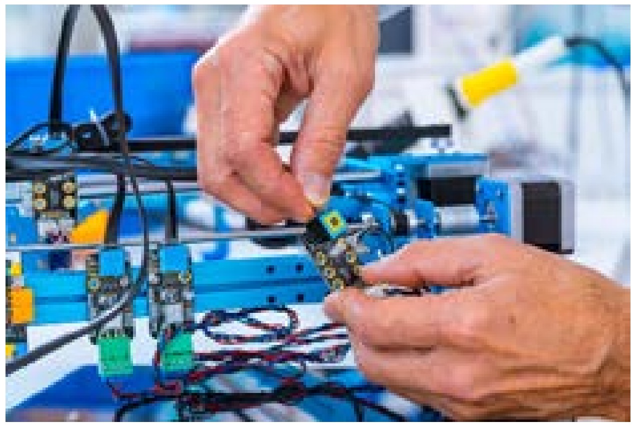
OR

Minimise $Z=x+2y$ subject to $2x+y\ge 3$, $x+2y\ge 6$, x, $y\ge 0$. Show that the minimum of Z occurs at more than two points.

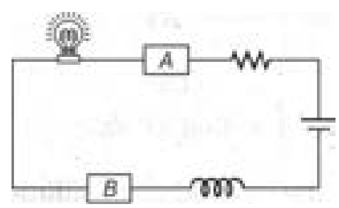
Section - E

Case study based questions are compulsory.

36. An electro-mechanical assembler basically makes machines or/and other assemblies that contain electronic components like wires or microchips. Typically assemblers use blueprints, work instructions, and computer software to manufacture whatever they are working on.



An electronic assembly consists of two sub-systems say A and B as shown below.



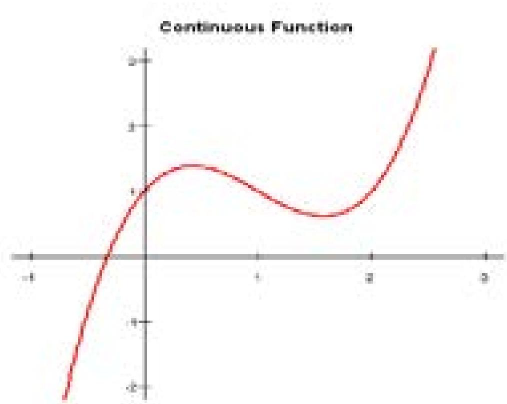
From previous testing procedures, the following probabilities are assumed to be known $P(A \text{ fails}) = 0.2$, $P(B \text{ fails alone}) = 0.15$, $P(A \text{ and } B \textit{ fail}) = 0.15$.

On the basis of above information, answer the following questions.

(i) Find the probability P(B fails) and the probability P(A fails alone).

(ii) Find the probability P(whole system fail) and the probability P(A fails/B has failed).

37. In mathematics, a continuous function is a function such that a continuous variation of the argument induces a continuous variation of the value of the function. This means that there are no abrupt changes in value, known as discontinuities.



Let $f(x)$ be a continuous function defined on $[a, b]$, then

$$\int_a^b f(x)\,dx = \int_0^b f(a+b-x)\,dx$$

On the basis of above information, answer the following questions.

(i) Evaluate $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\,dx$

(ii) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan x\,dx$

iii) Evaluate $\int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}\,dx]$

OR

Evaluate $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx$

38. Bob is taking a learning test in which the time he takes to memorize items from a given list is recorded. Let $M(t)$ be the number of items he can memorize in t minutes. His learning rate is found to be

$$M'(t) = 0.4t - 0.005t^2$$

(i) How many items can Bob memorize during the first 10 minutes?
(ii) How many additional items can he memorize during the next 10 minutes (from time $t = 10$ to $t = 20$)?

□□□□□□□

Sample Paper 5

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $f(x) = \begin{cases} \dfrac{\log x}{x-1}, & if\ x \neq 1 \\ k, & if\ x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is

(a) 0 (b) -1

(c) 1 (d) e

2. $\int \sqrt{1+\cos x}\, dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right) + C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right) + C$

(c) $2\sqrt{2}\sin\left(\frac{x}{2}\right) + C$ (d) $\frac{1}{2}\sin\left(\frac{x}{2}\right) + C$

3. If $\int \frac{x+2}{2x^2+6x+5}\, dx = P\int \frac{4x+6}{2x^2+6x+5}\, dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

4. If $\int_0^a f(2a-x)\, dx = m$ and $\int_0^a f(x)\, dx = n$, then $\int_0^{2a} f(x)\, dx$ is equal to

(a) $2m+n$ (b) $m+2n$

(c) $m-n$ (d) $m+n$

5. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

6. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit
(b) $\frac{2}{3}$ sq unit
(c) 1 sq unit
(d) $\frac{4}{3}$ sq unit

7. If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = i + j$, then A is equal to

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
(d) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

8. Discuss the continuity of the function $f(x) = \sin 2x - 1$ at the point $x = 0$ and $x = \pi$

(a) Continuous at $x = 0,\ \pi$
(b) Discontinuous at $x = 0$ but continuous at $x = \pi$
(c) Continuous at $x = 0$ but discontinuous at $x = \pi$
(d) Discontinuous at $x = 0,\ \pi$

9. Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$
(b) $\cos 3x$
(c) $\tan x$
(d) $\cos 2x$

10. For what values of x, function $f(x) = x^4 - 4x^3 + 4x^2 + 40$ is monotonic decreasing?

(a) $0 < x < 1$
(b) $1 < x < 2$
(c) $2 < x < 3$
(d) $4 < x < 5$

11. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$
(b) $\log 64$
(c) $\log 81$
(d) $\log 27$

12. The family of curves $y = e^{a \sin x}$, where a is an arbitrary constant is represented by the differential equation

(a) $\log y = \tan x \frac{dy}{dx}$
(b) $y \log y = \tan x \frac{dy}{dx}$
(c) $y \log y = \sin \frac{dy}{dx}$
(d) $\log y = \cos x \frac{dy}{dx}$

13. The solution of $\frac{dy}{dx} = \frac{ax + g}{by + f}$ represents a circle, when

(a) $a = b$
(b) $a = -b$

(c) $a = -2b$ (d) $a = 2b$

14. Order of the equation $\left(1+5\frac{dy}{dx}\right)^{3/2} = 10\frac{d^3y}{dx^3}$ is

(a) 2 (b) 3

(c) 1 (d) 0

15. The figure formed by four points $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}$, $3\hat{i}-5\hat{j}-2\hat{k}$, $\hat{k}-\hat{j}$ is a

(a) parallelogram (b) rectangle

(c) trapezium (d) square

16. If a line makes angles 90°, 60° and θ with X, Y and Z-axis respectively, where θ is acute angle, then find θ.

(a) 30^o (b) 60^o

(c) 45^o (d) 90^o

17. If $P(A \cup B) = 0.83$, $P(A) = 0.3$ and $P(B) = 0.6$, then the events will be

(a) dependent

(b) independent

(c) cannot say anything

(d) None of the above

18. Find the equation of the line which passes through the point $(1,2,3)$ and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$.

(a) $(\hat{i}+\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$

(b) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$

(c) $(\hat{i}+2\hat{j}+5\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$

(d) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda(5\hat{i}+2\hat{j}-2\hat{k})$

19. **Assertion :** area of the parallelogram whose adjacent sides are $\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}-j+\hat{k}$ is $3\sqrt{2}$ square units.
Reason : area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $|\vec{a}-\vec{b}|$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion:** if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, then $P(A \cup B)$ is $\frac{11}{26}$

Reason: J, F, E_1 and E_2 are two events. then

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cup E_2)}{P(E_2)},\ 0 < P(E_2) \leq 1$$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$. Write the values of $|B|$.

22. Determine the value of k for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

OR

Determine the value of constant k so that the function $f(x) = \begin{cases} \dfrac{kx}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

23. Find the general solution of the following differential equation $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

24. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

25. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Show that the relation S in the set R of real numbers defined as $S = \{(a, b): a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive not symmetric not transitive.

27. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the system of equations $x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

Continue on next page.....

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Use it to solve the system of equations

$$2x-3y+5z=11.$$
$$3x+2y-4z=-5,$$
$$x+y-2z=-3.$$

28. If the function $f(x)$ given by $f(x)=\begin{cases}3ax+b, & if\ x>1\\ 11, & if\ x=1\\ 5ax-2b, & if\ x<1\end{cases}$ is continuous at $x=1$, then find the values of a and b.

OR

Find the value of k, so that the functions f defined by $f(x)=\begin{cases}kx+1, & if\ x\leq\pi\\ \cos x & if\ x>\pi\end{cases}$ is continuous at $x=\pi$.

29. Evaluate $\int\frac{x^3+3x+4}{\sqrt{x}}$

30. Find the particular solution of the differential equation $(+e^{2x})\,dy+(1+y^2)\,e^x\,dx=0$, given that $y=1$, when $x=0$.

31. Find the coordinates of the foot of perpendicular drawn from the point $A(-1,8,4)$ to the line joining the points $B(0,-1,3)$ and $C(2,-3,-1)$. Hence, find the image of the point A in the line BC.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. (i) Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

(ii) Write the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

(iii) Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

Using the principal value, evaluate the following:

(i) $\tan^{-1}1+\sin^{-1}\left(-\frac{1}{2}\right)$

(ii) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

33. Find the intervals in which the function given by $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36x}{5}+11$ is

(i) strictly increasing
(ii) strictly decreasing.

OR

A ladder 5 m long is leaning against a wall. Bottom of ladder is pulled along the ground away from wall at the rate of 2 m/s. How fast is the height on the wall decreasing, when the foot of ladder is 4 m away from the wall?

34. Find the shortest distance between the following lines. $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$, $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

OR

Solve graphically the following system of in equations.
$x+2y\geq 20$, $3x+y\leq 15$

35. Maximise and minimise $Z=x+2y$ subject to the constraints

$$x+2y \geq 100$$
$$2x-y \leq 0$$
$$2x+y \leq 200$$
$$x,\ y \geq 0$$

Solve the above LPP graphically.

OR

A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer profit on an item of model A is Rs. 15 and on an items of model B is Rs. 10. How many of items of models should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.

Section - E

Case study based questions are compulsory.

36. Publishing is the activity of making information, literature, music, software and other content available to the public for sale or for free. Traditionally, the term refers to the creation and distribution of printed works, such as books, newspapers, and magazines.

NODIA Press is a such publishing house having two branch at Jaipur. In each branch there are three offices. In each office, there are 2 peons, 5 clerks and 3 typists. In one office of a branch, 5 salesmen are also working. In each office of other branch 2 head-clerks are also working. Using matrix notations find :

(i) the total number of posts of each kind in all the offices taken together in each branch.

(ii) the total number of posts of each kind in all the offices taken together from both branches.

Continue on next page.....

37. Different types of drugs affect your body in different ways, and the effects associated with drugs can vary from person to person. How a drug effects an individual is dependent on a variety of factors including body size, general health, the amount and strength of the drug, and whether any other drugs are in the system at the same time. It is important to remember that illegal drugs are not controlled substances, and therefore the quality and strength may differ from one batch to another.

The concentration $C(t)$ in milligrams per cubic centimeter (mg/cm^3) of a drug in a patient's bloodstream is $0.5\,\text{mg/cm}^3$ immediately after an injection and t minutes later is decreasing at the rate

$$C'(t) = \frac{-0.01e^{0.01t}}{(e^{0.01t}+1)^2}\ \text{mg/cm}^3 \text{ per minute}$$

A new injection is given when the concentration drops below $0.05\,\text{mg/cm}^3$.

(i) Find an expression for $C(t)$.

(ii) What is the concentration after 1 hour? After 3 hours?

38. The Vande Bharat Express, also known as Train 18, is a semi-high-speed, intercity, electric multiple unit train operated by the Indian Railways on 4 routes as of October 2022. Routes include New Delhi to Shri Mata Vaishno

In a survey at Vande Bharat Train, IRCTC asked the passenger to rate and review the food served in train. Suppose IRCTC asked 500 passenger selected at random to rate food according to price (low, medium, or high) and food (1, 2, 3, or 4 stars). The results of this survey are presented in the two-way, or contingency, table below. The numbers in this table represent frequencies. For example, in the third row and fourth column, 30 people rated the prices high and the food 4 stars. The last column contains the sum for each row, and similarly, the bottom row contains the sum for each column. These sums are often called marginal totals.

Continue on next page.....

Price	Food rating			
	*	**	***	****
Low	20	30	90	10
Medium	50	80	90	30
High	20	10	40	30

Assume that these results are representative of the entire passenger of train, so the relative frequency of occurrence is the true probability of the event. A passenger from train is randomly selected.

(i) Find the probability that the passenger rates the prices medium.

(ii) Find the probability that the passenger rates the food 2 stars.

(iii) Suppose the passenger selected rates the prices high. What is the probability that he rates the restaurants 1 star?

OR

(iv) Suppose the passenger selected does not rate the food 4 stars. What is the probability that she rates the prices high?

□□□□□□□

Sample Paper 6

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $A = \{1, 2, 3, 4\}, B = \{1, 2, 3\}$, then number of mappings from A to B is

(a) 3^4 (b) 12

(c) 4^3 (d) 2^7

2. The image of the interval $[1, 3]$ under the mapping $f : R \to R$ given by $f(x) = 2x^3 - 24x + 107$ is

(a) $[75, 89]$ (b) $[74, 89]$

(c) $[0, 75]$ (d) none of these

3. The value of $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$ is

(a) 15 (b) 5

(c) 13 (d) 14

4. If matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{bmatrix}$ is singular, then λ is equal to

(a) -2 (b) -1

(c) 1 (d) 2

5. If x is measured in degrees, then $\frac{d}{dx}(\cos x)$ is equal to

(a) $-\sin x$ (b) $-\frac{180}{\pi}\sin x$

(c) $-\frac{\pi}{180}\sin x$ (d) $\sin x$

6. Let $f(x)=x|x|$, then $f(x)$ has a

(a) local maxima at $x=0$

(b) local minima at $x=0$

(c) point of inflexion at $x=0$

(d) none of the above

7. The least, value of the function $f(x)=ax+b/x$, $a>0$, $b>0$, $x>0$ is

(a) $\sqrt{ab}$

(b) $2\sqrt{\frac{a}{b}}$

(c) $2\sqrt{\frac{b}{a}}$

(d) $2\sqrt{ab}$

8. $\int_0^{\pi/2}\left|\cos\left(\frac{x}{2}\right)\right|dx$ is equal to

(a) 1

(b) -2

(c) $\sqrt{2}$

(d) 0

9. The value of $\int_{-2}^{2}(x\cos x+\sin x+1)\,dx$ is

(a) 2

(b) 0

(c) -2

(d) 4

10. The area of the region bounded by the lines $y=mx$, $x=1$, $x=2$ and X-axis is 6 sq units, then m is equal to

(a) 3

(b) 1

(c) 2

(d) 4

11. The order of the differential equation whose general solution is given by $y=(c_1+c_2)\sin(x+c_3)-c_4e^{x+c^5}$, is

(a) 5

(b) 4

(c) 2

(d) 3

12. The solution of the differential equation $\frac{dy}{dx}+1e^{x+y}$

(a) $(x+c)\,e^{x+y}=0$

(b) $(x+y)\,e^{x+y}=0$

(c) $(x-c)\,e^{x+y}=1$

(d) $(x-c)\,e^{x+y}=0$

13. Solution of $\frac{dy}{dx}+y\sec x=\tan x$ is

(a) $y(\sec x+\tan x)=\sec x+\tan x-x+c$

(b) $y=\sec x+\tan x-x+c$

(c) $y(\sec x+\tan x)=\sec x+\tan x+x+c$

(d) none of the above

14. The vector $\vec{a}$ is equal to

(a) $(\vec{a}\cdot\hat{k})\hat{i}+(\vec{a}\cdot\hat{i})\hat{j}+(\vec{a}\cdot\hat{j})\hat{k}$

(b) $(\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k}$

(c) $(\vec{a}\cdot\hat{j})\hat{i}+(\vec{a}\cdot\hat{k})\hat{j}+(\vec{a}\cdot\hat{i})\hat{k}$

(d) $(\vec{a}\cdot\vec{a})(\hat{i}+\hat{j}+\hat{k})$

15. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors of magnitude 3, 4 and 5 respectively. If $\vec{a}$ is perpendicular to $(\vec{b}+\vec{c}), \vec{b}$ is perpendicular to $(\vec{c}+\vec{a})$ and $\vec{c}$ is perpendicular to$(\vec{a}+\vec{b})$, then the magnitude of the vector $\vec{a}+\vec{b}+\vec{c}$ is

(a) 5 (b) $5\sqrt{2}$

(c) $5\sqrt{3}$ (d) 2

16. The equation of the line through the point $(2, 3, -5)$ and equally inclined to the axes are

(a) $x-2 = y-3 = z+5$ (b) $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{-5}$

(c) $\frac{x}{2} = \frac{y}{3} = \frac{z}{-5}$ (d) none of these

17. A bag A contains 4 green and 3 red balls and bog B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(a) $\frac{2}{7}$ (b) $\frac{2}{3}$

(c) $\frac{3}{7}$ (d) $\frac{1}{3}$

18. Solution set of the inequality $x \geq 0$ is

(a) half plane on the left of y-axis

(b) half plane on the right of y-axis excluding the points of y-axis

(c) half plane on the right of y-axis including the points on y-axis

(d) none of the above

19. **Assertion:** Scalar matrix $A = [a_{ij}] = \begin{cases} k: & i = j \\ 0: & i \neq j \end{cases}$ where k is a scalar, in an identity matrix when $k = 1$

Reason: Every identity matrix is not a scalar matrix.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. Consider the function $f(x) = \begin{cases} x^2, & x \geq 1 \\ x+1, & x < 1 \end{cases}$

Assertion: f is not derivable at $x = 1$ as $\lim\limits_{x \to 1^{-1}} f(x) \neq \lim\limits_{x \to 1^{+}} f(x)$

Reason: If a function f is derivable at a point a then it is continuous at a

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class [0].

22. Evaluate $\int \frac{(\log x)^2}{x} dx$.

OR

Evaluate $\int \frac{e^{\tan^{-1}x}}{1 + x^2} dx$.

23. Write the projection of $(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

OR

If $\vec{a}$ and $\vec{b}$ are to vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, the prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

24. If a line has direction ratios $(2, -1, -2)$, then what are its direction cosines?

25. Two dice are thrown n times in succession. What is the probability of obtaining a doublet six atleast once?

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Solve the following equation for x.

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

27. Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

28. If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

29. Find the intervals in which the function given by $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is

(i) increasing (ii) decreasing.

OR

Show that of all the rectangles of given area, the square has the smallest perimeter.

30. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1 + e^x} dx$.

OR

Find $\int \frac{x^3}{x^4 + 3x^2 + 2} dx$.

31. If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=5$, $|\vec{b}|=6$ and $|\vec{c}|=9$, then find the angle between $\vec{a}$ and $\vec{b}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the vector and cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$$

and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.

33. Evaluate $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$.

34. Find the particular solution of the differential equation

$$x\frac{dy}{dx}-y+x\operatorname{cosec}\left(\frac{y}{x}\right)=0$$

OR

Find the particular solution of the differential equation

$$\frac{dy}{dx}=1+x+y+xy, \text{ given that } y=0 \text{ when } x=1.$$

35. Maximise $Z=5x+3y$ subject to the constraints: $3x+5y\le 15$; $5x+2y\le 10$, $x\ge 0$, $y\ge 0$.

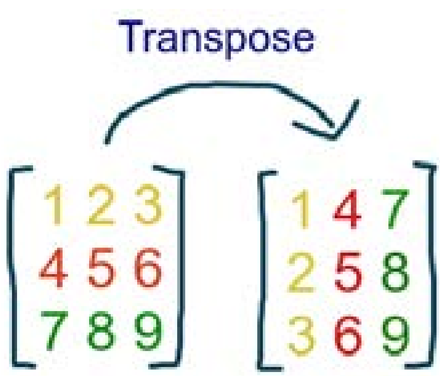
OR

Minimize $Z=3x+5y$ such that $x+3y\ge 3$, $x+y\ge 2$, x, $y\ge 0$.

Section - E

Case study based questions are compulsory.

36. In linear algebra, the transpose of a matrix is an operator which flips a matrix over its diagonal; that is, it switches the row and column indices of the matrix A by producing another matrix, often denoted by A^T. The transpose of a matrix was introduced in 1858 by the British mathematician Arthur Cayley.



If $A = [a_{ij}]$ be a $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. A square matrix $A = [a_{ij}]$ is said to be symmetric, $A^T = A$ for all possible values of i and j. A square matrix A $[a_{ij}]$ is said to be skew-symmetric, if $A^T = -A$ for all possible values of i and j. Based on the above, information, answer the following questions.

(i) Find the transpose of [1, -2, -5].

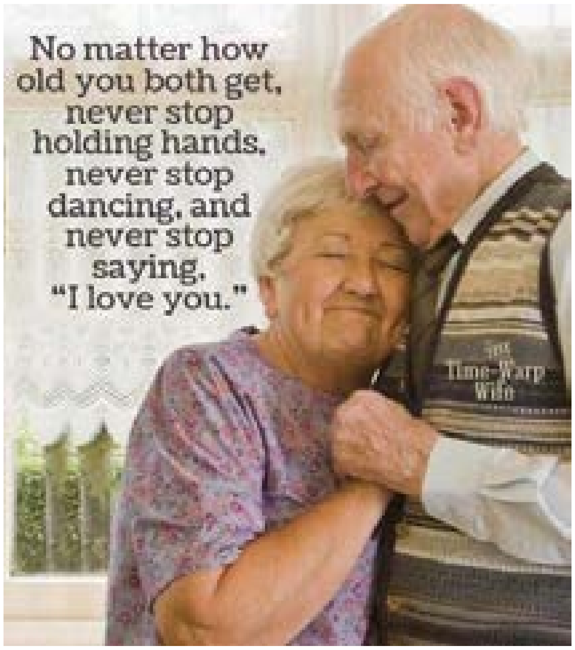
(ii) Find the transpose of matrix (ABC).

(iii) Evaluate $(A+B)^T - A,. A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

OR

Evaluate $(AB)^T$, where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

37. Measures of joint and survivor life expectancy are potentially useful to those designing or evaluating policies affecting older couples and to couples making retirement, savings, and long-term care decisions. However, couple-based measure of life expectancies are virtually unknown in the social sciences.



The odds against a husband who is 45 yr old, living till he is 70 are. 7 : 5 and the odds against his wife who is now 36, living till she is 61 are 5 : 3.

On the basis of above information, answer the following questions.

(i) Find the probabilities of husband living till 70 and wife living till 61.

(ii) Find the probability P(couple will be alive 25 yr hence).

(iii) Find the probability P(exactly one of them will be alive 25 yr hence).

OR

Find the probability P(none of them will be alive 25 yr hence) and probability P(atleast one of them will be alive 25 yr hence).

38. RK Verma is production analysts of a ready-made garment company. He has to maximize the profit of company using data available. He find that $P(x) = -6x^2 + 120x + 25000$ (in Rupee) is the total profit function of a company where x denotes the production of the company.

Based on the above information, answer the following questions.

(i) Find the profit of the company, when the production is 3 units.

(ii) Find $P'(5)$

(iii) Find the interval in which the profit is strictly increasing.

OR

Find the production, when the profit is maximum.

□□□□□□□

Sample Paper 7

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. The maximum value of xe^{-x} is

(a) e (b) $1/e$

(c) $-e$ (d) $-1/e$

2. For all real values of x, the minimum value of $\dfrac{1-x+x^2}{1+x+x^2}$ is

(a) 0 (b) 1

(c) 3 (d) $\frac{1}{3}$

3. Evaluate $\int \dfrac{x^3-x^2+x-1}{x-1}dx$

(a) $\frac{x^3}{3}+x+C$ (b) $\frac{x^3}{4}+x+C$

(c) $\frac{x^3}{5}+x+C$ (d) $\frac{x^3}{6}+x+C$

4. $\int \dfrac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}dx$ is equal to

(a) $\sin(\tan^{-1}x)+C$ (b) $\tan(\sec^{-1}x)+C$

(c) $\tan(\sin^{-1}x)+C$ (d) $-\tan(\cos^{-1}x)+C$

5. $\int \sqrt{1+\cos x}\,dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right)+C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

(c) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}\sin\left(\frac{x}{2}\right)+C$

1. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined, then B is of the type

(a) 4×4 (b) 3×4

(c) 4×3 (d) 3×3

2. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then

(a) $A^2 + 7A - 5I = O$ (b) $A^2 - 7A + 5I = O$

(c) $A^2 + 5A - 7I = O$ (d) $A^2 - 5A + 7I = O$

3. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and $Q = PP^T$, then the value of Q is

(a) 2 (b) -2

(c) 1 (d) 0

4. The derivative of $\cos^3 x$ with respect to $\sin^3 x$ is

(a) $-\cot x$ (b) $\cot x$

(c) $\tan x$ (d) $-\tan x$

5. If $\sin(x+y) = \log(x+y)$, then dy/dx is equal to

(a) 2 (b) -2

(c) 1 (d) -1

6. The area bounded by $y = \log x$, X-axis and ordinates $x = 1$, $x = 2$ is

(a) $\frac{1}{2}(\log 2)^2$ (b) $\log(2/e)$

(c) $\log(4/e)$ (d) $\log 4$

7. The area bounded by the parabola $y^2 = 8x$ and its latusrectum is

(a) 16/3 sq units (b) 32/3 sq units

(c) 8/3 sq units (d) 64/3 sq units

8. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3$, then the value of $|\vec{a} + \vec{b}|$ is

(a) equal to (b) greater than 1

(c) equal to (d) less than 1

9. If $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, then the angle between a and b is

(a) 45° (b) 180°

(c) 90° (d) 60°

10. The point of intersection of lines

$$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$$

and $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}$ is

(a) $(5,7,-2)$ (b) $(-3,3,6)$

(c) $(2,10,4)$ (d) $\left(21,\frac{5}{3},\frac{10}{3}\right)$

11. Equation of the line passing through $(2,-1,1)$ and parallel to the line $\frac{x-5}{4} = \frac{y+1}{-3} = \frac{z}{5}$ is

(a) $\frac{x-2}{4} = \frac{y+1}{-3} = \frac{z-1}{5}$ (b) $\frac{x-2}{4} = \frac{y+1}{3} = \frac{z-1}{5}$

(c) $\frac{x-2}{-4} = \frac{y+1}{-3} = \frac{z-1}{5}$ (d) None of the above

12. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

13. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

(a) $\frac{1}{72}$ (b) $\frac{3}{73}$

(c) $\frac{5}{72}$ (d) $\frac{1}{74}$

14. **Assertion :** The elimination of four arbitrary constants in $y = (c_1 + c_2 + c_3 e^{c_4})x$ results into a differential equation of the first order $x\frac{dy}{dx} = y$

Reason : Elimination of n arbitrary constants requires in general a differential equation of the n^{th} order.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true, reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

15. **Assertion :** The general solution of $\frac{dy}{dx} + y = 1$ is $ye^x = e^x + c$

Reason : The number of arbitrary constants is in the general solution of the differential equation is equal to the order of differential equation.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true, reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

16. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$.
Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

17. Find the general solution of the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

OR

Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

18. If a line has direction ratios $(2, -1, -2)$, then what are its direction cosines?

19. Minimize $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, x, $y \geq 0$.

20. Prove that if E and F are independent events, then the events E and F' are also independent.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

21. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$

22. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

OR

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the values of $(A^2 - 5A)$.

Continue on next page.....

23. A ladder 5 m is leaning against a wall .The bottom of the ladder is pulled along the ground anyway from the wall, at the rate 2 cm/s. How fast is its height on the wall decreasing, when the foot of the ladder is 4 meter away from the wall?

OR

A balloon, which always remains spherical on inflation, is being inflated by pumping is 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

24. Find $\int \frac{2\cos x}{(1-\sin x)(2-\cos^2 x)}dx$.

25. Find the vector and cartesian equations of the line through the point $(1, 2, -4)$ and perpendicular to the lines

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$$

26. Solve graphically the following system of in equations.
$x + 2y \geq 20$, $3x + y \leq 15$

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

27. Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that $R = \{(a, b): a, b \in A, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

Let $A = R - \{2\}$, $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \left(\frac{x-1}{x-2}\right)$, show that f is one-one and onto. Hence find f^{-1}

28. If $y = x^x$, then prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

OR

For what value of λ, the function defined by $f(x) = \begin{cases} \lambda(x^2+2), & if\ x \leq 0 \\ 4x+6, & x > 0 \end{cases}$ is continuous at $x = 0$?

Hence, check the differentiability of $f(x)$ at $x = 0$.

29. Using integration, find the area of the following region.
$\{(x, y) : |x-1| \leq y \leq \sqrt{5 - x^2}\}$

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

OR

Using integration, find the area of the following region. $\left\{(x, y) : \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\right\}$

Continue on next page.....

30. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find unit vector along $\vec{b} \times \vec{c}$.

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of the same magnitude, then prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Section - E

Case study based questions are compulsory.

31. When it comes to taxes, there are two types of taxes in India - Direct and Indirect tax. The direct tax includes income tax, gift tax, capital gain tax, etc while indirect tax includes goods and service tax i.e. GST and any local tax.

A company earns before tax profits of ₹100000. It is committed to making a donation to the Red Cross 10% of its after-tax profits. The Central Government levies income taxes of 50% of profits after deducing charitable donations and any local taxes. The company must also pay local taxes of 10% of its profit less the donation to the Red Cross.

(i) Taking the suitable variable form the system of equation that represent given problem.

(ii) Compute how much the company pays in income taxes, local taxes and as a donation to the Red Cross, using Cramer's Rule.

32. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially $(t = 0)$ 50 g of the first substance was present; 1 hr later, only 10 g of it remained.

(i) Find an expression that gives the amount of the first substance present at any time t.

(ii) What is the amount present after 2 hr?

33. Joint Entrance Examination – Advanced, is an academic examination held annually in India. It is organised by one of the seven zonal IITs under the guidance of the Joint Admission Board on a round-robin rotation pattern for the qualifying candidates of the JEE-Main. A candidate can attempt JEE (Advanced) maximum of two times in two consecutive years. A successful candidates get the admission in any IITs of India based on merit.

Applicants have a 0.26 probability of passing IIT advanced test when they take it for the first time, and if they pass it they can get admission in IIT. However, if they fail the test the first time, they must take the test a second time, and when applicants take the test for the second time there is a 0.43 chance that they will pass and be allowed to get admission. Applicants are rejected if the test is failed on the second attempt.

(i) What is the probability that an applicant gets admission in IIT but needs two attempts at the test?

(ii) What is the probability that an applicant gets admission in IIT?

(iii) If an applicant gets admission in IIT, what is the probability that he or she passed the test on the first attempt?

□□□□□□□

Sample Paper 8

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. Let $f(x) = x^3 + \frac{3}{2}x^2 + 3x + 3$, then $f(x)$ is

(a) am even function (b) an odd function

(c) an increasing function (d) a decreasing function

2. The length of the largest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

3. $\int \frac{dx}{x(x^7+1)}$ is equal to

(a) $\log\left(\frac{x^7}{x^7+1}\right) + C$ (b) $\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right) + C$

(c) $\log\left(\frac{x^7+1}{x^7}\right) + C$ (d) $\frac{1}{7}\log\left(\frac{x^7+1}{x^7}\right) + C$

4. The value of $\int_0^1 \frac{dx}{e^x + e}$ is

(a) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$ (b) $\log\left(\frac{1+e}{2}\right)$

(c) $\frac{1}{e}\log(1+e)$ (d) $\log\left(\frac{2}{1+e}\right)$

5. The area bounded by the parabola $y^2 = 8x$ and its latus rectum is

(a) 16/3 sq units (b) 32/3 sq units

(c) 8/3 sq units (d) 64/3 sq units

6. If $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-one function from A into B is

(a) 1340 (b) 1860

(c) 1430 (d) 1680

7. The relation $\operatorname{cosec}^{-1}\left(\frac{x^2+1}{2x}\right) = 2\cot^{-1}x$ is valid for

(a) $x \geq 1$ (b) $x \geq 0$

(c) $|x| \geq 1$ (d) none of these

8. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and $Q = PP^T$, then the value of Q is

(a) 2 (b) -2

(c) 1 (d) 0

9. If $x = e^{y + e^{y + e^{y + \dots}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x}$ (b) $\frac{1-x}{x}$

(c) $\frac{x}{1+x}$ (d) None of these

10. The point of discontinuous of $\tan x$ are

(a) $n\pi,\ n \in I$ (b) $2n\pi,\ n \in I$

(c) $(2n+1)\frac{\pi}{2},\ n \in I$ (d) None of these

11. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$

(a) 1 (b) 3

(c) 2 (d) none of these

12. The solution of the differential equation $x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x}$ is

(a) $\tan\frac{y}{2x} = c - \frac{1}{2x^2}$ (b) $\tan\frac{y}{x} = c + \frac{1}{x}$

(c) $\cos\frac{y}{x} = 1 + \frac{c}{x}$ (d) $x^2 = (c + x^2)\tan\frac{y}{x}$

13. Equation to the curve through (2,1) whose slope at the point (x, y) is $\frac{x^2+y^2}{2xy}$, is

(a) $2(x^2 - y^2) = 3x$ (b) $2(y^2 - x^2) = 6y$

(c) $x(x^2 - y^2) = 6$ (d) none of these

14. If $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$, then $\vec{a}, \vec{b}$ are

(a) perpendicular

(b) like parallel

(c) unlike parallel

(d) coincident

15. Let G be the centroid of a triangle ABC and O be any other point, then $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ is equal to

(a) $\vec{0}$

(b) $\overrightarrow{OG}$

(c) $3\overrightarrow{OG}$

(d) none of these

16. The projections of a line segment on x, y, z axes are 12, 4, 3. The length and the direction cosines of the line segment are

(a) $13, < \frac{12}{13}, \frac{4}{13}, \frac{3}{13} >$

(b) $19, < \frac{12}{19}, \frac{4}{19}, \frac{3}{19} >$

(c) $11, < \frac{12}{11}, \frac{14}{11}, \frac{3}{11} >$

(d) none of these

17. If A and B are mutually exclusive events with $P(B) \neq 1$, then $P(A/\overline{B})$ is equal to (here, $\overline{B}$ is the complement of the event B).

(a) $\frac{1}{P(B)}$

(b) $\frac{1}{1 - P(B)}$

(c) $\frac{P(A)}{P(B)}$

(d) $\frac{P(A)}{1 - P(B)}$

18. Two dice are thrown n times in succession. The probability of obtaining a doublet six atleast once is

(a) $\left(\frac{1}{36}\right)^n$

(b) $1 - \left(\frac{35}{36}\right)^n$

(c) $\left(\frac{1}{12}\right)^n$

(d) None of these

19. **Assertion:** If $A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}$, then orders of $(A + B)$ is 2×3

Reason: If $[aij]$ and $[bij]$ are two matrics of the same order, then order of $A + B$ is the same as the order of A or B

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** The relation R in a set $A = \{1,2,3,4\}$ defined by $R = \{(x,y): 3x - y = 0)\}$ have the domain $= \{1,2,3,4\}$ and range $= \{3,6,9,12\}$

Reason: Domain and range of the relation (R) is respectively the set of all first & second entries of the distinct ordered pair of the relation.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Show that the relation R on IR defined as $R = \{(a,b): (a \le b)\}$, is reflexive and transitive but not symmetric.

22. Write the value of $\int \frac{2 - 3\sin x}{\cos^2 x} dx$.

OR

Write the value of $\int \sec x(\sec x + \tan x)\, dx$.

23. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Write the value of λ, so that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

24. The equation of a line is

$$5x - 3 = 15y + 7 = 3 = 3 - 10z$$

Write the direction cosines of the line.

25. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

What is the value of p ?

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\}\ \tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$

27. If $\begin{vmatrix} 3x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.

28. If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

29. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x, throughout its domain.

OR

Find the intervals in which the function given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is
(i) increasing
(ii) decreasing.

30. Evaluate $\int_0^{\pi} |x^3 - x| dx$.

OR

Evaluate $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$.

31. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the values of p, so that the lines

$$l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$$

and $$l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

33. Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

34. Solve the following differential equation $\operatorname{cosec} x \log|y| \frac{dy}{dx} + x^2 y^2 = 0$.

OR

Solve the following differential equation.

$$x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x;\ x \neq 0$$

35. Maximize $Z = 3x + 4y$, subject to the constraints; $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

OR

Minimize $Z = -3x + 4y$ subject to the constraints

$$\begin{aligned} x + 2y &\leq 8, \\ 3x + 2y &\leq 12, \\ x &\geq 0,\ y \geq 0 \end{aligned}$$

Section - E

Case study based questions are compulsory.

36. RK Verma is production analysts of a ready-made garment company. He has to maximize the profit of company using data available. He find that $P(x) = -6x^2 + 120x + 25000$ (in Rupee) is the total profit function of a company where x denotes the production of the company.

Based on the above information, answer the following questions.

(i) Find the profit of the company, when the production is 3 units.

(ii) Find $P'(5)$

(iii) Find the interval in which the profit is strictly increasing.

OR

Find the production, when the profit is maximum.

37. Mahesh runs a form processing agency. He collect form from different office and then extract data and record data on computer. In his office three employees Vikas, Sarita and Ishaan process incoming copies of a form. Vikas process 50% of the forms. Sarita processes 20% and Ishaan the remaining 30% of the forms. Vikas has an error rate of 0.06, Sarita has an error rate of 0.04 and Ishaan has an error rate of 0.03.

Based on the above information answer the following questions.

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vikas.

38. Publishing is the activity of making information, literature, music, software and other content available to the public for sale or for free. Traditionally, the term refers to the creation and distribution of printed works, such as books, newspapers, and magazines.

NODIA Press is a such publishing house having two branch at Jaipur. In each branch there are three offices. In each office, there are 2 peons, 5 clerks and 3 typists. In one office of a branch, 5 salesmen are also working. In each office of other branch 2 head-clerks are also working. Using matrix notations find :

(i) the total number of posts of each kind in all the offices taken together in each branch.

(ii) the total number of posts of each kind in all the offices taken together from both branches.

□□□□□□□

Sample Paper 9

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If gas is being pumped into a spherical balloon at the rate of 30 ft^3/min. Then, the rate at which the radius increases, when it reaches the value 15 ft is

(a) $\frac{1}{15\pi}$ ft/min (b) $\frac{1}{30\pi}$ ft/min

(c) $\frac{1}{20}$ ft/min (d) $\frac{1}{25}$ ft/min

2. The length of the longest interval, in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

3. If $\int \frac{x+2}{2x^2+6x+5}dx = P\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

4. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then the matrix $A^2(\alpha)$ is equal to

(a) $A(2\alpha)$ (b) $A(\alpha)$

(c) $A(3\alpha)$ (d) $A(4\alpha)$

5. If $f(x) = \begin{cases} ax+3, & x \le 2 \\ a^2x-1, & x > 2 \end{cases}$, then the values of a for which f is continuous for all x are

(a) 1 and -2
(b) 1 and 2
(c) -1 and 2
(d) -1 and -2

6. For the value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x < 0 \end{cases}$$

continuous at $x = 0$?

(a) 0
(b) 1
(c) 2
(d) Not continuous at $x = 0$ for any value of λ

7. $\int_0^{\pi/2} \left|\cos\left(\frac{x}{2}\right)\right| dx$ is equal to

(a) 1
(b) -2
(c) $\sqrt{2}$
(d) 0

8. Area of the region satisfying $x \leq 2$, $y \leq |x|$ and $x \geq 0$ is

(a) 4 sq units
(b) 1 sq unit
(c) 2 sq units
(d) None of these

9. The area bounded by the parabola $y^2 = 8x$ and its latusrectum is

(a) 16/3 sq units
(b) 32/3 sq units
(c) 8/3 sq units
(d) 64/3 sq units

10. Solution of $e^{dy/dx} = x$, when $x = 1$ and $y = 0$ is

(a) $y = x(\log x - 1) + 4$
(b) $y = x(\log x - 1) + 3$
(b) $y = x(\log x + 1) + 1$
(d) $y = x(\log x - 1) + 1$

11. Solution of the differential equation $xdy - ydx = 0$ represents a

(a) parabola
(b) circle
(c) hyperbola
(d) straight line

12. An integrating factor of the differential equation $x\frac{dy}{dx} + y \log x = xe^x x^{-\frac{1}{2}\log x} (x > 0)$ is

(a) $x^{\log x}$
(b) $(\sqrt{x})^{\log x}$
(c) $(\sqrt{e})^{(\log x)^2}$
(d) e^{x^2}

13. The figure formed by four points $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}$, $3\hat{i}-5\hat{j}-2\hat{k}$, $\hat{k}-\hat{j}$ is a

(a) parallelogram (b) rectangle

(c) trapezium (d) square

14. If $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}$ makes an angle of 60° with $\vec{a}$, then

(a) $|\vec{a}|=2|\vec{b}|$ (b) $2|\vec{a}|=|\vec{b}|$

(c) $|\vec{a}|=\sqrt{3}|\vec{b}|$ (d) $\sqrt{3}|\vec{a}|=|\vec{b}|$

15. If the direction cosines of a line are $\left(\frac{1}{c},\frac{1}{c},\frac{1}{c}\right)$, then

(a) $0<c<1$ (b) $c>2$

(c) $c=\pm\sqrt{2}$ (d) $c=\pm\sqrt{3}$

16. Find the angle between the following pairs of lines:

$$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$$

and $$\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$$

(a) $\cos^{-1}\left(\frac{19}{21}\right)$ (b) $\cos^{-1}\left(\frac{16}{21}\right)$

(c) $\cos^{-1}\left(\frac{13}{21}\right)$ (d) $\cos^{-1}\left(\frac{11}{21}\right)$

17. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

18. If $P(A)=\frac{1}{12}$, $P(B)=\frac{5}{12}$ and $P\left(\frac{B}{A}\right)=\frac{1}{15}$, then $P(A\cup B)$ is equal to

(a) $\frac{89}{180}$ (b) $\frac{90}{180}$

(c) $\frac{91}{180}$ (d) $\frac{92}{180}$

19. **Assertion :** $\int_{-\pi/2}^{\pi/2}|\sin x|\,dx=2$

Reason : $\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$, where c $\varepsilon\, a, b)$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true, reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. If $n > 1$, then

Assertion : $\int_0^\infty \frac{dx}{1+x^n} = \int_0^1 \frac{dx}{(1-x^n)^{1/n}}$

Reason : $\int_a^b f(x)\,dx = \int_a^b f(a+b+x)\,dx$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true, reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

OR

For what value of x, $A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ is a singular matrix?

22. Determine the value of constant k so that the function $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$.

23. Find the general solution of equation $\frac{dy}{dx} + y = 1 (y \neq 1)$

24. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

25. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Check whether the relation R in the set R of real numbers, defined by $R = \{(a, b) : 1 + ab > 0\}$, is reflexive, symmetric or transitive.

27. If $A=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2=4A-3I$. Hence, find A^{-1}.

OR

Show that for the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, $A^3-6A^2+5A+11I=O$. Hence, find A^{-1}.

28. Find the value of k, for which $f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if -1\leq x<0\\ \dfrac{2x+1}{x-1}, & if\ 0\leq x<1\end{cases}$ is continuous at $x=0$.

OR

If $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & when\ x<0\\ a, & when\ x=0\\ \dfrac{\sqrt{x}}{\sqrt{17+\sqrt{x}}-4}, & when\ x>0\end{cases}$ and f is continuous at $x=0$, then find the value of a.

29. Find $\int\frac{x-3}{(x-1)^3}e^x\,dx$.

30. Find the general solution of equation $y\log y\,dx-x\,dy=0$

31. Find the position of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of line segment RQ.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Write the following functions in the simplest from:

$$\tan^{-1}\frac{\sqrt{1+x^2}}{x},\ x\neq 0$$

OR

Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Thinking Process

Use the property, $\tan^{-1}\tan x=x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\cos^{-1}(\cos x)=x,\ x\in[0,\pi]$ to get the answer.

33. AB is the diameter of a circle and C is any point of the circle. Show that the area of ΔABC is maximum, when it is an isosceles triangle.

OR

Find the minimum value of $(ax+by)$, where $xy=c^2$.

34. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

OR

Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

and $$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}.$$

35. Solve graphically.
$x + y \geq 5$, $2x + 3 \geq 3y$, $0 \leq x \leq 4$, $0 \leq y \leq 2$.

OR

Solve the following system of inequalities graphically:
$3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$.

Section - E

Case study based questions are compulsory.

36. Fertilizer, natural or artificial substance containing the chemical elements that improve growth and productiveness of plants. Fertilizers enhance the natural fertility of the soil or replace chemical elements taken from the soil by previous crops.

Continue on next page.....

The following matrix gives the proportionate mix of constituents used for three fertilisers:

$$\begin{array}{c} \\ \text{I} \\ \text{Fertilisers II} \\ \text{III} \end{array} \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \quad \text{Constituents} \\ \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix} \end{array}$$

(i) If sales are 1000 tins (of one kilogram) per week, 20% being fertiliser I, 30% being fertiliser II and 50% being fertiliser III, how much of each constituent is used.

(ii) If the cost of each constituents is ₹ 5, ₹ 6, ₹ 7.5 and ₹ 10 per 100 grams, respectively, how much does a one kilogram tin of each fertiliser cost

(iii) What is the total cost per week?

37. Bata India is the largest retailer and leading manufacturer of footwear in India and is a part of the Bata Shoe Organization. Incorporated as Bata Shoe Company Private Limited in 1931, the company was set up initially as a small operation in Konnagar (near Calcutta) in 1932. In January 1934,

The manager of BATA show room at Jaipur determines that the price p (dollars) for each pair of a popular brand of sports sneakers is changing at the rate of

$$p'(x) = \frac{-300x}{(x^2+9)^{3/2}}$$

when x (hundred) pairs are demanded by consumers. When the price is \$75 per pair, 400 pairs $(x=4)$ are demanded by consumers.

(i) Find the demand (price) function $p(x)$.

(ii) At what price will 500 pairs of sneakers be demanded? At what price will no sneakers be demanded?

(iii) How many pairs will be demanded at a price of \$90 per pair?

Continue on next page.....

38. Goods and Services Tax (GST) is an indirect tax (or consumption tax) used in India on the supply of goods and services. It is a comprehensive, multistage, destination-based tax: comprehensive because it has subsumed almost all the indirect taxes except a few state taxes. Multi-staged as it is, the GST is imposed at every step in the production process, but is meant to be refunded to all parties in the various stages of production other than the final consumer and as a destination-based tax, it is collected from point of consumption and not point of origin like previous taxes.

A GST form is either filed on time or late, is either from a small or a large business, and is either accurate or inaccurate. There is an 11% probability that a form is from a small business and is accurate and on time. There is a 13% probability that a form is from a small business and is accurate but is late. There is a 15% probability that a form is from a small business and is on time. There is a 21% probability that a form is from a small business and is inaccurate and is late.

(i) If a form is from a small business and is accurate, what is the probability that it was filed on time?

(ii) What is the probability that a form is from a large business?

□□□□□□□

Sample Paper 10

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $f(x)=\begin{cases}\dfrac{\log x}{x-1}, & if\ x\neq 1\\ k, & if\ x=1\end{cases}$ is continuous at $x=1$, then the value of k is

(a) 0 (b) -1

(c) 1 (d) e

2. Minimum value of the function $f(x)=x^2+x+1$ is

(a) 1 (b) 3

(c) $\frac{3}{4}$ (d) 4

3. The function $f(x)=2+4x^2+6x^4+8x^6$ has

(a) only one maxima (b) only one minima

(c) no maxima and minima (d) many maxima and minima

4. $\int\frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$ is equal to

(a) $\log(x^e+e^x)+C$ (b) $e\log(x^e+e^x)+C$

(c) $\frac{1}{e}\log(x^e+e^x)+C$ (d) None of these

5. Area of the region satisfying $x\leq 2,\ y\leq|x|$ and $x\geq 0$ is

(a) 4 sq units (b) 1 sq unit

(c) 2 sq units (d) None of these

6. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then which of the following relations is a function from A to B?

(a) $\{(1, 2), (2, 3), (3, 4), (2, 2)\}$

(b) $\{(1, 2), (2, 3), (1, 3)\}$

(c) $\{(1, 3), (2, 3), (3, 3)\}$

(d) $\{(1, 1), (2, 3), (3, 4)\}$

7. Range of the function $f(x) = \dfrac{x}{1 + x^2}$ is

(a) $(-\infty, \infty)$

(b) $[-1, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(d) $[-\sqrt{2}, \sqrt{2}]$

8. $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1 - x^2})$ holds good for all

(a) $|x| \le 1$

(b) $1 \ge x \ge 0$

(c) $|x| \le \dfrac{1}{\sqrt{2}}$

(d) none of these

9. The existence of the unique solution of the system of equations $x + y + z = \beta$; $5x - y + \alpha z = 10$ and $2x + 3y = 6$ depends on

(a) α only

(b) β only

(c) Both α and β

(d) Neither β nor α

10. At $x = \dfrac{3}{2}$, the function $f(x) = \dfrac{|2x - 3|}{2x - 3}$ is

(a) continuous

(b) discontinuous

(c) differentiable

(d) non-zero

11. The order of the differential equation $\left[1 + \left(\dfrac{dy}{dx}\right)\right]^{3/2} = \dfrac{d^2y}{dx^2}$ is

(a) 1

(b) 2

(c) 3

(d) 4

12. The integrating factor of the differential equation $\dfrac{dy}{dx}(x\log x) + y = 2\log x$ is given by

(a) e^x

(b) $\log x$

(c) $\log\log x$

(d) x

13. If $dy/dx = e^{-2y}$ and $y = 0$, when $x = 5$, then the value of x, when $y = 3$ is

(a) e^5

(b) $e^6 + 1$

(c) $\dfrac{e^6 + 9}{2}$

(d) $\log_e 6$

14. Given two vectors
$\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, $\vec{b} = -2\hat{i} + 2\hat{j} - \hat{k}$ and $\lambda = \dfrac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projectin of } \vec{b} \text{ on } \vec{a}}$ then the value of λ is

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$

(c) 3 (d) 7

15. A constant force $\vec{F} = 2\vec{i} - 3\vec{j} + 2\vec{k}$ is acting on a particle such that the particle is displaced from the point $A(1, 2, 3)$ to the point $B(3, 4, 5)$. The work done by the force is

(a) 2 (b) 3

(c) $\sqrt{17}$ (d) $2\sqrt{51}$

16. The points (5, 2, 4), $(6, -1, 2)$ and $(8, -7, k)$ are collinear, if k is equal to

(a) -2 (b) 2

(c) 3 (d) -1

17. If x-coordinate of a point P of line joining the points and $R(5, 2, -2)$ is 4, then the z-coordinate of P is

(a) -2 (b) -1

(c) 1 (d) 2

18. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then $P(A \cup B)$ is equal to

(a) $\frac{89}{180}$ (b) $\frac{90}{180}$

(c) $\frac{91}{180}$ (d) $\frac{92}{180}$

19. **Assertion:** $f(\theta) \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ = \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ is independent of θ

Reason: If $f(\theta) = c$ then $f(\theta)$ is independent of θ

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan x}\, dx = \frac{\pi}{\sqrt{2}}$

Reason: $\tan x = t^2$ makes the integrand in I as a rational function.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

22. What is the range of the function

$$f(x) = \frac{|x-1|}{x-1},\ x \neq 1?$$

23. Write the value of $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$.

OR

Write the value of $\int \frac{dx}{x^2 + 16}$.

24. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$.

OR

If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, then find the projection of $\vec{a}$ on $\vec{b}$.

25. If a line makes angles 90°, 60° and θ with X, Y and Z-axis respectively, where θ is acute angle, then find θ.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$.

27. Write the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

28. In the interval $\frac{\pi}{2} < x < \pi$, find the value of x for which the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular.

29. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

30. The sides of an equilateral triangle are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when the side is 10 cm?

OR

Find the value(s) of x for which $y = [x(x-2)]^2$ is an increasing function.

31. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$.

OR

Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$$

and $$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k}).$$

OR

Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

33. Evaluate $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$.

34. Solve the following differential equation. $\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$

OR

Find the particular solution of the differential equation $(3xy + y^2)\,dx + (x^2 + xy)\,dy = 0$, for $x = 1$ and $y = 1$.

35. Maximise and minimise $Z = x + 2y$ subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x,\ y \geq 0$$

Solve the above LPP graphically.

OR

Maximise $Z = 8x + 9y$ subject to the constraints given below

$$2x + 3y \leq 6$$
$$3x - 2y \leq 6$$
$$y \leq 1$$
$$x,\ y \geq 0.$$

Section - E

Case study based questions are compulsory.

36. At its simplest, a fair die states that each of the faces has a similar probability of landing facing up. A standard fair six-sided die, for example, can be regarded as "fair" if each of the faces consists of a probability of 1/6.

A fair die is rolled. Consider the events $A = \{1,3,5\}$, $B = \{2,3\}$, and $C = \{2,3,4,5,\}$

On the basis of above information, answer the following questions.

(i) Find the probability $P(A/B)$ and $P(B/A)$.

(ii) Find the probability $P(A/C)$, $P(A \cap B/C)$ and $P(A \cup B/C)$

37. Sachin Mehara is a final year student of civil engineering at IIT Delhi. As a final year real time project, he has got the job of designing a auditorium for cultural activities purpose. The shape of the floor of the auditorium is rectangular and it has a fixed perimeter, say. P.

Based on the above information, answer the following questions.

(i) If l and b represents the length and breadth of the rectangular region, then find the relationship between l, b, p.

(ii) Find the area (A) of the floor, as a function of is l

(iii) College authority is interested in maximising the area of the floor A. For this purpose, find the value of l.

OR

Find the maximum area of the floor.

38. Pfizer Inc. is an American multinational pharmaceutical and biotechnology corporation headquartered on 42nd Street in Manhattan, New York City. The company was established in 1849 in New York by two German immigrants, Charles Pfizer and his cousin Charles F. Erhart. Pfizer develops and produces medicines and vaccines for immunology, oncology, cardiology, endocrinology, and neurology.

The purchase officer of the Pfizer informs the production manger that during the month, following supply of three chemicals, Asprin (A), Caffieine (C) and Decongestant (D) used in the production of three types of pain-killing tablet will be 16, 10 and 16 kg respectively. According to the specification, each strip of 10 tables of Paingo requires 2 gm of A, 3 gm of C and 1 gm of D. The requirements for other tables are:

X-prene	4 gm of A	1 gm of C	3 gm of D
Relaxo	1 gm of A	2 gm of C	3 gm of D

(i) Taking the suitable variable form the system of equation that represent given problem.

(ii) Use matrix inversion method to find the number of strips of each type so that raw materials are consumed entirely.

□□□□□□□

Sample Paper 11

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

2. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x} dx$ is equal to

(a) $-\log(1+\sin^2 x) + C$ (b) $\log(1+\cos^2 x) + C$

(c) $-\log(1+\cos^2 x) + C$ (d) $\log(1+\tan^2 x) + C$

3. The value of $\int_{-2}^{2} (x\cos x + \sin x + 1)\, dx$ is

(a) 2 (b) 0

(c) -2 (d) 4

4. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

5. If A and B are two equivalence relations defined on set C, then

(a) $A \cap B$ is an equivalence relation (b) $A \cap B$ is not an equivalence relation

(c) $A \cap B$ is an equivalence relation (d) $A \cap B$ is not an equivalence relation

6. If A and B are two symmetric matrices of same order. Then, the matrix $AB - BA$ is equal to

(a) a symmetric matrix
(b) a skew-symmetric matrix
(c) a null matrix
(d) the identity matrix

7. If $x = e^{y + e^{y + e^{y + \dots}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x}$
(b) $\frac{1-x}{x}$
(c) $\frac{x}{1+x}$
(d) None of these

8. The derivative of $\log|x|$ is

(a) $\frac{1}{x}, x > 0$
(b) $\frac{1}{|x|}, x \neq 0$
(c) $\frac{1}{x}, x \neq 0$
(d) None of these

9. The condition that $f(x) = ax^3 + bx^2 + cx + d$ has no extreme value is

(a) $b^2 > 3ac$
(b) $b^2 = 4ac$
(c) $b^2 = 3ac$
(d) $b^2 < 3ac$

10. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit
(b) $\frac{2}{3}$ sq unit
(c) 1 sq unit
(d) $\frac{4}{3}$ sq unit

11. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle, when

(a) $a = b$
(b) $a = -b$
(c) $a = -2b$
(d) $a = 2b$

12. The family of curves $y = e^{a \sin x}$, where a is an arbitrary constant is represented by the differential equation

(a) $\log y = \tan x \frac{dy}{dx}$
(b) $y \log y = \tan x \frac{dy}{dx}$
(c) $y \log y = \sin \frac{dy}{dx}$
(d) $\log y = \cos x \frac{dy}{dx}$

13. The order of the differential equation whose solution is $y = a \cos x + b \sin x + ce^{-x}$, is

(a) 3
(b) 1
(c) 2
(d) 4

14. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to

(a) 16
(b) 8
(c) 3
(d) 12

15. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\vec{x} \cdot \vec{y} = 0$, then

(a) $|\vec{x} + \vec{y}| = 1$
(b) $|\vec{x} + \vec{y}| = \sqrt{3}$
(c) $|\vec{x} + \vec{y}| = 2$
(d) $|\vec{x} + \vec{y}| = \sqrt{2}$

16. The distance of the plane $6x - 3y + 2z - 14 = 0$ from the origin is

(a) 2
(b) 1
(c) 14
(d) 8

17. If $P(A \cup B) = 0.83$, $P(A) = 0.3$ and $P(B) = 0.6$, then the events will be

(a) dependent
(b) independent
(c) cannot say anything
(d) None of the above

18. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$
(b) $\frac{1}{6}$
(c) $\frac{5}{6}$
(d) $\frac{1}{3}$

19. Let us define $\tan^{-1}\text{A} + \tan^{-1}\text{B} = \tan^{-1}\left(\frac{\text{A}+\text{B}}{1-\text{AB}}\right)$

Assertion : The value of $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$.

Reason : If $x > 0, y > 0$ than $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right) = \frac{\pi}{4}$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.
(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.
(c) Assertion is true, reason is false.
(d) Assertion is false, reason is true.

20. **Assertion :** If A is a matrix of order 2×2, then $|\text{adj A}| = |\text{A}|$
Reason : $|\text{A}| = |\text{A}^{\text{T}}|$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.
(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.
(c) Assertion is true, reason is false.
(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Write the vector equation of a line passing through point $(1,-1,2)$ and parallel to the line whose equation is $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$.

22. Write the value of $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$.

23. Find the differential equation representing the family of curves $y=ae^{bx+5}$, where a and b are arbitrary constants.

24. Write a unit vector in the direction of the sum of the vectors $\vec{a}=2\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-7\hat{k}$.

OR

If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors.

25. Two groups are computing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product introduced way by the second group.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $A=\{1,2,3\}, B=\{4,5,6,7\}$ and
$f=\{(1,4),(2,5),(3,6)\}$ is a function from A to B. State whether f is one-one or not.

27. Solve for x, $\cos^{-1}x+\sin^{-1}\left(\frac{x}{2}\right)=\frac{\pi}{6}$.

28. Given $A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$, compute A^{-1} and show that $2A^{-1}=9I-A$.

OR

If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $A^{-1}=kA$, then find the value of k.

29. Evaluate $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$.

30. Evaluate $\int_0^{\pi/2} x^2\sin x\,dx$.

OR

Evaluate $\int_{\pi/4}^{\pi/2} \cos 2x\cdot\log(\sin x)\,dx$.

31. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then find the values of a and b.

OR

Find the adjoint of the matrix $A=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and hence show that $A(adj\,A)=|A|I_3$.

33. If $x=\cos t+\log\tan\left(\frac{t}{2}\right)$, $y=\sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$.

OR

Find the values of a and b, if the function f defined by $f(x)=\begin{cases}x^2+3x+a, & x\le 1\\ bx+2, & x>1\end{cases}$ is differentiable at $x=1$.

34. Find the particular solution of the differential equation satisfying the given condition.

$$x^2\,dy+(xy+y^2)\,dx=0,\text{ when } y(1)=1$$

OR

Find the particular solution of the differential equation

$$x\frac{dy}{dx}-y+x\,\text{cosec}\left(\frac{y}{x}\right)=0$$

35. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}\cdot\vec{c}=18$.

Section - E

Case study based questions are compulsory.

36. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20% of the population is accident prone.

On the basis of above information, answer the following questions.

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policy holder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

37. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially ($t = 0$) 50 g of the first substance was present; 1 hr later, only 10 g of it remained.

(i) Find an expression that gives the amount of the first substance present at any time t.

(ii) What is the amount present after 2 hr?

38. Vitamins are nutritional substances which you need in small amounts in your diet. Vitamins A and E are fat-soluble vitamins, meaning they're stored in your body's fat cells, but they need to have their levels topped up regularly. Vitamin C is a water-soluble vitamin found in citrus and other fruits and vegetables, and also sold as a dietary supplement. It is used to prevent and treat scurvy. Vitamin C is an essential nutrient involved in the repair of tissue, the formation of collagen, and the enzymatic production of certain neurotransmitters.

A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C, while food II contains 1 unit per kg of vitamin A an 2 units per kg of vitamin C. It costs Rs. 30 per kg to purchase food I and Rs. 42 per kg to purchase food II.

(i) Formulate above as an LPP and solve it graphically.

(ii) Find the minimum cost of such a mixture.

□□□□□□□

Sample Paper 12

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. The point of discontinuous of $\tan x$ are

(a) $n\pi,\ n \in I$

(b) $2n\pi,\ n \in I$

(c) $(2n+1)\frac{\pi}{2},\ n \in I$

(d) None of these

2. If $y = a\log|x| + bx^2 + x$ has its extreme value at $x = -1$ and $x = 2$, then

(a) $a = 2, b = -1$

(b) $a = 2, b = -\frac{1}{2}$

(c) $a = -2, b = \frac{1}{2}$

(d) $a = -2, b = -\frac{1}{2}$

3. The point of inflexion for the curve $y = x^{5/3}$ is

(a) $(1,1)$

(b) $(0,0)$

(c) $(1,0)$

(d) $(0,1)$

4. If $\int \frac{x+2}{2x^2+6x+5}dx = P\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 2

5. $\int_0^{\pi/2} \left|\cos\left(\frac{x}{2}\right)\right| dx$ is equal to

(a) 1

(b) -2

(c) $\sqrt{2}$

(d) 0

6. If A and B are two equivalence relations defined on set C, then

(a) $A \cap B$ is an equivalence relation

(b) $A \cap B$ is not an equivalence relation

(c) $A \cap B$ is an equivalence relation

(d) $A \cap B$ is not an equivalence relation

7. The range of the function $f(x) = x^2 + 2x + 2$ is

(a) $(1, \infty)$ (b) $(2, \infty)$

(c) $(0, \infty)$ (d) $[1, \infty)$

8. If $x, y, z \varepsilon R$ and $x + y + z = xyz$, then the value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{4}$

9. Find the values of x, y and z from the following equations $\begin{bmatrix} 4 & x - z \\ 2 + y & xz \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -1 & 10 \end{bmatrix}$.

(a) $x = -5,\ y = 3,\ z = 2$ (b) $x = 5,\ y = -3,\ z = 2$

(c) $x = 5,\ y = 3,\ z = -2$ (d) $x = 5,\ y = -3,\ z = -2$

10. The value of k such that the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is

(a) 20 (b) -7

(c) 7 (d) -20

11. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit (b) $\frac{2}{3}$ sq unit

(c) 1 sq unit (d) $\frac{4}{3}$ sq unit

12. The solution of the equation $(x^2 - xy)\,dy = (xy + y^2)\,dx$ is

(a) $xy = ce^{-y/x}$ (b) $xy = ce^{-x/y}$

(c) $yx^2 = ce^{1/x}$ (d) none of these

13. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(a) straight line (b) circle

(c) parabola (d) ellipse

14. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$ is

(a) $x+y=\frac{x^2}{2}+c$

(b) $x-y=\frac{1}{3}x^3+c$

(c) $xy=\frac{1}{4}x^4+c$

(d) $y-x=\frac{1}{4}x^4+c$

15. Let D, E, F are the mid points of sides BC, CA, AB respectively of $\triangle ABC$. Which of the following is true?

(a) $\overrightarrow{AB}=2\overrightarrow{ED}$

(b) $\overrightarrow{AB}=2\overrightarrow{DE}$

(c) $\overrightarrow{AB}=\overrightarrow{ED}$

(d) $\overrightarrow{AB}=2\overrightarrow{DF}$

16. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30° and 120°, then $|\vec{a}\times\vec{b}|$ equals

(a) $\sqrt{\frac{2}{3}}$

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) 2

17. The points (5, 2, 4), $(6,-1,2)$ and $(8,-7,k)$ are collinear, if k is equal to

(a) -2

(b) 2

(c) 3

(d) -1

18. If A and B are two independent events such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$

(b) $\frac{1}{6}$

(c) $\frac{5}{6}$

(d) $\frac{1}{3}$

19. Consider, if $u=f(n)$, $v=g(x)$, then the derivative of f with respect to g is $\frac{du}{dv}=\frac{du/dx}{dv/dx}$.

Assertion: Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is 1 for $0<x<1$

Reason: $\sin^{-1}\left(\frac{2x}{1+x^2}\right)\neq\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $-1\leq 1x\leq 1$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, then $P(A \cup B)$ is $\frac{11}{26}$

Reason: E_1 and E_2 are two events. then $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cup E_2)}{P(E_2)}$, $0 < P(E_2) \leq 1$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $R = \{(x, y): x + 2y = 8\}$ is a relation on N, then write the range of R.

22. Evaluate $\int \frac{2}{1 + \cos 2x} dx$.

OR

Write the value of $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$.

23. Find λ and μ, if $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

OR

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then write the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

24. If a line makes angles 90° and 60°, respectively with the positive directions of X and Y-axes, find the angle which it makes with the positive direction of Z-axis.

25. Prove that if E and F are independent events, then the events E and F' are also independent.

Section - C

This section comprises of short answer-type questions (SA) of 5 marks each.

26. Write the value of $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$.

27. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$, write the value of $|AB|$.

28. If $y = e^{\tan^{-1} x}$, prove that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$.

29. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(i) strictly increasing.

(ii) strictly decreasing.

OR

Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36x}{5} + 11$ is

(i) strictly increasing

(ii) strictly decreasing.

30. Evaluate $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.

OR

Evaluate $\int \frac{dx}{x(x^5 + 3)}$.

31. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find $\int \frac{4}{(x-2)(x^2+4)} dx$.

OR

Find $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$.

33. A wire of length 28 m is to be cut into two pieces. One of the two pieces is to be made into a square and the other into a circle. What should be the lengths of two pieces, so that the combined area of circle and square is minimum?

OR

Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

34. Find the angle between following pair of lines.

$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.

OR

Find the shortest distance between lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}.$$

35. An urn contains 4 white and 6 red balls. Four balls are drawn at random (without replacement) from the urn. Find the probability distribution of the number of white balls?

OR

Find the probability distribution of number of doublets in three tosses of a pair of dice.

Section - E

Case study based questions are compulsory.

36. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially $(t = 0)$ 50 g of the first substance was present; 1 hr later, only 10 g of it remained.

(i) Find an expression that gives the amount of the first substance present at any time t.

(ii) What is the amount present after 2 hr?

37. Rice is a nutritional staple food which provides instant energy as its most important component is carbohydrate (starch). On the other hand, rice is poor in nitrogenous substances with average composition of these substances being only 8 per cent and fat content or lipids only negligible, i.e., 1 per cent and due to this reason it is considered as a complete food for eating. Rice flour is rich in starch and is used for making various food materials.

Two farmers Ramkishan and Gurcharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in ₹) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September Sales (in ₹)

$$\begin{array}{cccc} & \text{Basmati} & \text{Permal} & \text{Naura} \end{array}$$
$$A = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \begin{matrix} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{matrix}$$

October Sales (in ₹)

$$B = \begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array} \end{array}$$

(i) Find the combined sales in September and October for each farmer in each variety.
(ii) Find the decrease in sales from September to October.
(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

38. Global Air Lines has contracted with a tour group to transport a minimum of 1,600 first-class passengers and 4,800 economy-class passengers from New York to London during a 6-month time period. Global Air has two types of airplanes, the Orville 606 and the Wilbur W-1112. The Orville 606 carries 20 first-class passengers and 80 economy-class passengers and costs $12,000 to operate. The Wilbur W-1112 carries 80 first-class passengers and 120 economy-class passengers and costs $18,000 to operate.

During the time period involved, Global Air can schedule no more than 52 flights on Orville 606s and no more than 30 flights on Wilbur W-1112s.
(i) How should Global Air Lines schedule its flights to minimize its costs?
(ii) What operating costs would this schedule entail?

□□□□□□□

Sample Paper 13

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If x is real, then minimum value of $x^2 - 8x + 17$ is

(a) -1 (b) 0

(c) 1 (d) 2

2. If $\int_0^a f(2a-x)\,dx = m$ and $\int_0^a f(x)\,dx = n$, then $\int_0^{2a} f(x)\,dx$ is equal to

(a) $2m+n$ (b) $m+2n$

(c) $m-n$ (d) $m+n$

3. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to

(a) -1 (b) 0

(c) 1 (d) 2

4. If $x = \frac{2\,at}{1+t^3}$ and $y = \frac{2\,at^2}{(1+t^3)^2}$, then $\frac{dy}{dx}$ is equal to

(a) ax (b) a^2x^2

(c) $\frac{x}{a}$ (d) $\frac{x}{2a}$

5. If $f(x) = \begin{cases} \frac{3\sin\pi x}{5x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$

(c) $\frac{3\pi}{2}$

(d) $\frac{3\pi}{5}$

6. A sphere increases its volume at the rate of π cm³/s. The rate at which its surface area increases, when the radius is 1 cm is

(a) 2π sq cm/s

(b) π sq cm/s

(c) $\frac{3\pi}{2}$ sq cm/s

(d) $\frac{\pi}{2}$ sq cm/s

7. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π

8. $3a\int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to

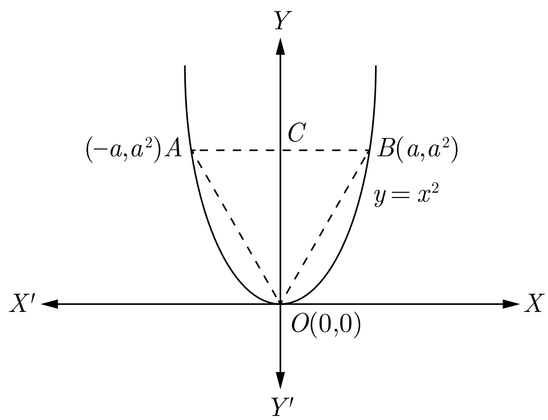
(a) $a - 1 + (a-1)^{-2}$

(b) $a + a^{-2}$

(c) $a - a^2$

(d) $a^2 + \frac{1}{a^2}$

9. The given figure shows a ΔAOB and the parabola $y = x^2$. The ratio of the area of the ΔAOB to the area of the region AOB of the parabola $y = x^2$ is equal to



(a) $\frac{3}{5}$

(b) $\frac{3}{4}$

(c) $\frac{7}{8}$

(d) $\frac{5}{6}$

10. The area bounded by $y = |\sin x|$, X-axis and the lines $|x| = \pi$ is

(a) 2 sq units

(b) 3 sq units

(c) 4 sq units

(d) None of these

11. Order of the equation $\left(1+5\frac{dy}{dx}\right)^{3/2}=10\frac{d^3y}{dx^3}$ is

(a) 2 (b) 3

(c) 1 (d) 0

12. $y=2e^{2x}-e^{-x}$ is a solution of the differential equation

(a) $y_2+y_1+2y=0$ (b) $y_2-y_1+2y=0$

(c) $y_2+y_1=0$ (d) $y_2-y_1-2y=0$

13. The projection of $\vec{a}=3\hat{i}-\hat{j}+5\hat{k}$ on $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$ is

(a) $\frac{8}{\sqrt{35}}$ (b) $\frac{8}{\sqrt{39}}$

(c) $\frac{8}{\sqrt{14}}$ (d) $\sqrt{14}$

14. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. Then, which one of the following is correct?

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=0$

(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq 0$

(c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=0$

(d) $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$ are mutually perpendicular.

15. The angle between the lines $x=1,\ y=2$ and $y=-1,\ z=0$ is

(a) 30° (b) 60°

(c) 90° (d) 0°

16. The line joining the points (1,1,2) and (3, – 2,1) meets the planes $3x+2y+z=6$ at the point

(a) (1,1,2) (b) (3, – 2,1)

(c) (2, – 3,1) (d) (3,2,1)

17. For two events A and B, if $P(A)=P\left(\frac{A}{B}\right)=\frac{1}{4}$ and $P\left(\frac{B}{A}\right)=\frac{1}{2}$, then

(a) A and B are independent events

(b) $P\left(\frac{A'}{B}\right)=\frac{3}{4}$

(c) $P\left(\frac{B'}{A}\right)=\frac{1}{2}$

(d) All of the above

Continue on next page.....

18. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$, then $P\left(\frac{A'}{B}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

19. **Assertion :** The matrix $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonel matrix.

Reason : If A and B are orthagonal, then AB is also orthegonal.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $\text{adj}(\text{adj}\, A) = A$.

Reason : $|\text{adj}(\text{adj}\, A)| = |A|^{(n-1)^2}$, where A be n rowed non-singular matrix.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Show that the function $f(x) = x^3 - 3x^2 + 3x$, $x \in R$ is increasing on R.

22. Find the general solution of following equation $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

23. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

OR

The x-coordinate of point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate.

24. Maximize $Z = x + y$,
subject to $x - y \leq -1$, $x + y \leq 0$, x, $y \geq 0$.

25. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Show that the modulus function $f: R \to R$, given by $f(x) = [x]$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

OR

Show that the Signum function of $f: R \to R$, given by

$$f(x) = \begin{cases} 1, & if \ x > 0 \\ 0, & if \ x = 0 \\ -1, & if \ x < 0 \end{cases}$$

is neither one-one nor onto.

27. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

28. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.

29. Find $\int \frac{(2x-5)\,e^{2x}}{(2x-3)^3}\,dx$.

OR

Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx$.

30. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

31. Maximise and minimise $Z = x + 2y$ subject to the constraints

$$\begin{aligned} x + 2y &\geq 100 \\ 2x - y &\leq 0 \\ 2x + y &\leq 200 \\ x, y &\geq 0 \end{aligned}$$

Solve the above LPP graphically.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b) R (c, d)$ if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

OR

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

33. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $x^2 \leq 1$, then find dy/dx.

OR

Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

34. Using integration, find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$.

OR

Using integration, find the area of the triangular region whose have the equation $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

35. Using vectors, find the area of the $\Delta\,ABC$, whose vertices are $A(1,2,3)$, $B(2,-1,4)$ and $C(4,5,-1)$.

OR

If lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$ intersect, then find the value of k and hence, find the equation of the plane containing these lines.

Section - E

Case study based questions are compulsory.

36. Pastry is a dough of flour, water and shortening that may be savoury or sweetened. Sweetened pastries are often described as bakers' confectionery. The word "pastries" suggests many kinds of baked products made from ingredients such as flour, sugar, milk, butter, shortening, baking powder, and eggs.

The Sunrise Bakery Pvt Ltd produces three basic pastry mixes A, B and C. In the past the mix of ingredients has shown in the following matrix:

$$\text{Type}\ \begin{matrix} A \\ B \\ C \end{matrix}\overset{\begin{matrix}\text{Flour} & \text{Fat} & \text{Sugar}\end{matrix}}{\begin{bmatrix} 5 & 1 & 1 \\ 6.5 & 2.5 & 0.5 \\ 4.5 & 3 & 2 \end{bmatrix}}$$

(All quantities in kg)

Due to changes in the consumer's tastes it has been decided to change the mixes using the following amendment matrix:

$$\text{Type}\ \begin{matrix} A \\ B \\ C \end{matrix}\overset{\begin{matrix}\text{Flour} & \text{Fat} & \text{Sugar}\end{matrix}}{\begin{bmatrix} 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \end{bmatrix}}$$

Continue on next page.....

OR

Using matrix algebra you are required to calculate:

(i) the matrix for the new mix:

(ii) the production requirement to meet an order for 50 units of type A, 30 units of type B and 20 units of type C of the new mix;

(iii) the amount of each type that must be made to totally use up 370 kg of flour, 170 kg of fat and 80 kg of sugar that are at present in the stores.

37. Brine is a high-concentration solution of salt in water. In diverse contexts, brine may refer to the salt solutions ranging from about 3.5% up to about 26%. Brine forms naturally due to evaporation of ground saline water but it is also generated in the mining of sodium chloride.

A tank initially contains 10 gallons of pure water. Brine containing 3 pounds of salt per gallon flows into the tank at a rate of 2 gallons per minute, and the well-stirred mixture flows out of the tank at the same rate.

(i) How much salt is present at the end of 10 minutes?

(ii) How much salt is present in the long run?

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38. ICAR-Indian Agricultural Research Institute is an autonomous body responsible for co-ordinating agricultural education and research in India. It reports to the Department of Agricultural Research and Education, Ministry of Agriculture. The Union Minister of Agriculture serves as its president. It is the largest network of agricultural research and education institutes in the world.

ICAR grows vegetables and grades each one as either good or bad for its taste, good or bad for its size, and good or bad for its appearance. Overall 78% of the vegetables have a good taste. However, only 69% of the vegetables have both a good taste and a good size. Also, 5% of the vegetables have both a good taste and a good appearance, but a bad size. Finally, 84% of the vegetables have either a good size or a good appearance.

(i) If a vegetable has a good taste, what is the probability that it also has a good size?

(ii) If a vegetable has a bad size and a bad appearance, what is the probability that it has a good taste?

□□□□□□□

Sample Paper 14

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $f(x)=\begin{cases}\dfrac{\log x}{x-1}, & if\ x\neq 1\\ k, & if\ x=1\end{cases}$ is continuous at $x=1$, then the value of k is

(a) 0 (b) -1

(c) 1 (d) e

2. If $f(x)=\begin{cases}\dfrac{3\sin\pi x}{5x}, & x\neq 0\\ 2k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$

(c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{5}$

3. The function $f(x)=x^2e^{-x}$ is strictly increases in the interval

(a) $(0,2)$ (b) $(0,\infty)$

(c) $(-\infty,0]\cup[2,\infty)$ (d) none of theses

4. If the function $f(x)=kx^3-9x^2+9x+3$ is monotonically increasing in every interval, then

(a) $k<3$ (b) $k\leq 3$

(c) $k>3$ (d) $k\geq 3$

5. $\int\frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}dx$ is equal to

(a) $\sin(\tan^{-1}x)+C$ (b) $\tan(\sec^{-1}x)+C$

(c) $\tan(\sin^{-1}x)+C$ (d) $-\tan(\cos^{-1}x)+C$

6. If $\sin^{-1}x = \theta + \beta$ and $\sin^{-1}y = \theta - \beta$, then $1 + xy$ is equal to

(a) $\sin^2\theta + \sin^2\beta$

(b) $\sin^2\theta + \cos^2\beta$

(c) $\cos^2\theta + \cos^2\beta$

(d) none of these

7. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = \hat{j}$. The value of $\vec{c}$ for which $\vec{a}, \vec{b}, \vec{c}$ form a right handed system is

(a) $\vec{0}$

(b) $z\hat{i} - x\vec{k}$

(c) $-z\hat{i} + x\hat{k}$

(d) $y\hat{j}$

8. If R is a relation on the set N, defined by $\{(x, y): 2x - y = 10\}$, then R is

(a) reflexive

(b) symmetric

(c) transitive

(d) None of the above

9. The domain of the function $f(x) = \sqrt{\cos x}$ is

(a) $\left[\frac{3\pi}{2}, 2\pi\right]$

(b) $\left[0, \frac{\pi}{2}\right]$

(c) $[-\pi, \pi]$

(d) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

10. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

11. If $\int_0^a f(2a - x)\,dx = m$ and $\int_0^a f(x)\,dx = n$, then $\int_0^{2a} f(x)\,dx$ is equal to

(a) $2m + n$

(b) $m + 2n$

(c) $m - n$

(d) $m + n$

12. The area bounded by $y = \log x$, X-axis and ordinates $x = 1$, $x = 2$ is

(a) $\frac{1}{2}(\log 2)^2$

(b) $\log(2/e)$

(c) $\log(4/e)$

(d) $\log 4$

13. Order of the differential equation of the family of all concentric circles centred at (h, k), is

(a) 2

(b) 3

(c) 1

(d) 4

14. If m and n are the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^5 + 4\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1, \text{ then}$$

(a) $m = 3, n = 2$
(b) $m = 3, n = 3$
(c) $m = 3, n = 5$
(d) $m = 3, n = 1$

15. The vectors $\vec{a} = 2\hat{i} - 3\hat{j}$ and $\vec{b} = -4\hat{i} + 6\hat{j}$ are

(a) coincident
(b) parallel
(c) perpendicular
(d) neither parallel nor perpendicular

16. A straight line which makes an angle of 60° with each of y and z axes, inclined with x-axis at an angle of

(a) 30°
(b) 45°
(c) 75°
(d) 60°

17. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$
(b) $\frac{1}{6}$
(c) $\frac{1}{3}$
(d) $\frac{2}{3}$

18. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$
(b) $\frac{3}{4}$
(c) $\frac{1}{4}$
(d) $\frac{2}{3}$

19. Let A be a 2×2 matrix.

Assertion: adj (adj A) $= A$.

Reason: $|\text{adj } A| = |A|$.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

20. **Assertion:** The equation of curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$

Reason: The solution of differential equation $\left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)(e^x + e^{-x}) + 1 = 0$ is $y = c_1 e^x + c_2 e^{-x}$.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

22. Evaluate $\int (1 - x)\sqrt{x}\, dx$.

OR

Given, $\int e^x (\tan x + 1) \sec x\, dx = e^x f(x) + C$. Write $f(x)$ satisfying above.

23. Write the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

OR

Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

24. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

25. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

27. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2nd row.

28. If $y = x\cos(a + y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$

Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$.

OR

If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

29. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm?

OR

The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing, when the side of the triangle is 20 cm?

30. Find $\int \frac{dx}{5 - 8x - x^2}$.

31. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one items is chosen at random from this and is found to be defective. What is the probability that it was produced by A?

OR

An insurance company insured 2000 scooter driver, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver?

33. Find $\int \frac{2\cos x}{(1 - \sin x)(2 - \cos^2 x)} dx$.

OR

Find $\int \frac{2\cos x}{(1 - \sin x)(1 + \sin^2 x)} dx$.

34. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

35. Find the values of p, so that the lines

$$l_1 : \frac{1 - x}{3} = \frac{7y - 14}{p} = \frac{z - 3}{2} \text{ and}$$

$$l_2 : \frac{7 - 7x}{3p} = \frac{y - 5}{1} = \frac{6 - z}{5}$$

are perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

Section - E

Case study based questions are compulsory.

36. A craftswoman produces two products: floor lamps and table lamps. Production of one floor lamp requires 75 minutes of her labor and materials that cost \$25. Production of one table lamp requires 50 minutes of labor, and the materials cost \$20. The craftswoman wishes to work no more than 40 hours each week, and her financial resources allow her to pay no more than \$900 for materials each week.

(i) If she can sell as many lamps as she can make and if her profit is \$39 per floor lamp and \$33 per table lamp, how many floor lamps and how many table lamps should she make each week to maximize her weekly profit?

(ii) What is that maximum profit?

37. The Indian toy industry is estimated to be worth US\$1.5 billion, making up 0.5% of the global market share. The toy manufacturers in India can mostly be found in NCR, Mumbai, Karnataka, Tamil Nadu, and several smaller towns and cities across central states such as Chhattisgarh and Madhya Pradesh. The sector is fragmented with 90% of the market being unorganised. The toys industry has been predicted to grow to US\$2-3 billion by 2024. The Indian toy industry only represents 0.5% of the global industry size indicating a large potential growth opportunity for Indian consumer product companies who will develop exciting innovations to deliver international quality standards at competitive prices.

Fisher Price is a leading toy manufacturer in India. Fisher Price produces x set per week at a total cost of $\frac{1}{25}x^2 + 3x + 100$. The produced quantity for his market is $x = 75 - 3p$ where p is the price set.

(i) Show that the maximum profit is obtained when about 30 toys are produced per week.
(ii) What is the price at maximum profit?

38. A manufacturing company has two service departments, S_1, S_2 and four production departments P_1, P_2, P_3 and P_4.
Overhead is allocated to the production departments for inclusion in the stock valuation. The analysis of benefits received by each department during the last quarter and the overhead expense incurred by each department were:

Service Department	**Percentages to be allocated to departments**					
	S_1	S_2	P_1	P_2	P_3	P_4
S_1	0	20	30	25	15	10
S_2	30	0	10	35	20	5
Direct overhead expense ₹ '000	20	40	25	30	20	10

You are required to find out following using matrix method.
(i) Express the total overhead of the service departments in the form of simultaneous equations.
(ii) Express these equations in a matrix form and solve for total overhead of service departments using matrix inverse method.
(iii) Determine the total overhead to be allocated from each of S_1 and S_2 to the production department.

□□□□□□□

Sample Paper 15

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $f(x) = \log_e(\sin x)$, then $f'(e)$ is equal to

(a) e^{-1} (b) e

(c) 1 (d) 0

2. The least, value of the function $f(x) = ax + b/x,\ a > 0,\ b > 0,\ x > 0$ is

(a) $\sqrt{ab}$ (b) $2\sqrt{\frac{a}{b}}$

(c) $2\sqrt{\frac{b}{a}}$ (d) $2\sqrt{ab}$

3. The point on the curve $x^2 = 2y$ which is nearest to the point $(0,5)$ is

(a) $(2\sqrt{2},4)$ (b) $(2\sqrt{2},0)$

(c) $(0,0)$ (d) $(2,2)$

4. The symmetric part of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 0 & -2 & -1 \\ -2 & 0 & -2 \\ -1 & -2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 3 \\ 4 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

5. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) 1

(d) -1

6. The value of $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$ is

(a) $\frac{x^2}{2}+\log|x|-2x+C$

(b) $\frac{x^2}{2}+\log|x|+2x+C$

(c) $\frac{x^2}{2}-\log|x|-2x+C$

(d) None of these

7. The area of the region bounded by the lines $y=mx$, $x=1$, $x=2$ and X-axis is 6 sq units, then m is equal to

(a) 3

(b) 1

(c) 2

(d) 4

8. Find the area of a curve $xy=4$, bounded by the lines $x=1$ and $x=3$ and X-axis.

(a) $\log 12$

(b) $\log 64$

(c) $\log 81$

(d) $\log 27$

9. The area of enclosed by $y=3x-5$, $y=0$, $x=3$ and $x=5$ is

(a) 12 sq units

(b) 13 sq units

(c) $13\frac{1}{2}$ sq units

(d) 14 sq units

10. The degree of the differential equation
$x=1+\left(\frac{dy}{dx}\right)+\frac{1}{2!}\left(\frac{dy}{dx}\right)^2+\frac{1}{3!}\left(\frac{dy}{dx}\right)^3+...$, is

(a) 3

(b) 2

(c) 1

(d) not defined

11. The solution of $\frac{dy}{dx}=\frac{ax+g}{by+f}$ represents a circle, when

(a) $a=b$

(b) $a=-b$

(c) $a=-2b$

(d) $a=2b$

12. The general solution of the differential equation $\frac{dy}{dx}=e^y(e^x+e^{-x}+2x)$ is

(a) $e^{-y}=e^x-e^{-x}+x^2+C$

(b) $e^{-y}=e^{-x}-e^x-x^2+C$

(c) $e^{-y}=-e^{-x}-e^x-x^2+C$

(d) $e^y=e^{-x}+e^x+x^2+C$

13. If $\lambda(3\hat{i}+2\hat{j}-6\hat{k})$ is a unit vector, then the value of λ is

(a) $\pm\frac{1}{7}$ (b) ± 7

(c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

14. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a}\cdot\vec{b}=|\vec{b}|^2$ and $|\vec{a}-\vec{b}|=\sqrt{7}$, then $|\vec{b}|$ is equal to

(a) $\sqrt{7}$ (b) $\sqrt{3}$

(c) 7 (d) 3

15. The direction cosines of the line joining the points $(4,3,-5)$ and $(-2,1,-8)$ are

(a) $\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$ (b) $\left(\frac{2}{7},\frac{3}{7},\frac{-6}{7}\right)$

(c) $\left(\frac{6}{7},\frac{3}{7},\frac{2}{7}\right)$ (d) None of these

16. If the lines $\frac{1-x}{3}=\frac{y-2}{2\alpha}=\frac{z-3}{2}$ and $\frac{x-1}{3\alpha}=y-1=\frac{6-z}{5}$ are perpendicular, then the value of α is

(a) $\frac{-10}{7}$ (b) $\frac{10}{7}$

(c) $\frac{-10}{11}$ (d) $\frac{10}{11}$

17. A bag A contains 4 green and 3 red balls and bog B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(a) $\frac{2}{7}$ (b) $\frac{2}{3}$

(c) $\frac{3}{7}$ (d) $\frac{1}{3}$

18. If $P(A)=\frac{4}{5}$, and $Q(A\cap B)=\frac{7}{10}$, then $P\left(\frac{B}{A}\right)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

19. **Assertion :** $\int\frac{xe^x}{(x+1)^2}dx=\frac{e^x}{x+1}+C$

Reason : $\int e^x\{f(x)+f'(x)\}\,dx=e^xf(x)+C$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** $\int \frac{dx}{e^x + e^{-x} + 2} = \frac{1}{e^x + 1} + C$

Reason : $\int \frac{d\{f(x)\}}{\{f(x)\}^2} = -\frac{1}{f(x)} + C$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

22. Differentiate $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$ with respect to x.

23. Find the general solution of differential equation $y = e^{2x}(a + bx)$

24. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

25. Suppose a girls throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once gets notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4 ,5 or 6 with the die?

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Let R be a relation defined on the set of natural numbers N as follow:

$$R = \{(x, y): x \in N, y \in N \; and \; 2x + y = 24\}$$

Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

27. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.

OR

Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(adj\ A) = |A| I_3$.

28. Find the values of a and b such that the function defined as follows is continuous.

$$f(x) = \begin{cases} x+2, & x \le 2 \\ ax+b, & 2 < x < 5 \\ 3x-2, & x \ge 5 \end{cases}$$

29. Find $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$.

OR

Integrate w.r.t. x, $\frac{x^2-3x+1}{\sqrt{1-x^2}}$.

30. Solve the following differential equation

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

31. Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the value of the following $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

OR

Find the value of the following $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

33. Find both the maximum value and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0,3]$.

OR

At what points in the interval $[0,2\pi]$, does the function $\sin 2x$ atain its maximum value?

34. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

OR

Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$$

and $$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k}).$$

35. Maximize $Z = -x + 2y$, Subject to the constraints:

$$x \ge 3,\ x + y \ge 5,\ x + 2y \ge 6,\ y \ge 0$$

OR

Minimise $Z = x + 2y$ subject to $2x + y \ge 3$, $x + 2y \ge 6$, x, $y \ge 0$. Show that the minimum of Z occurs at more than two points.

Section - E

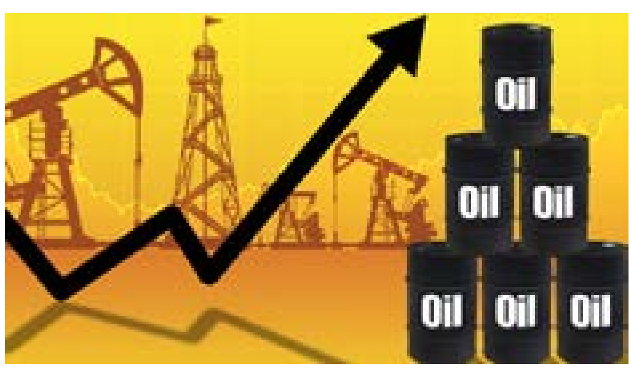
Case study based questions are compulsory.

36. Sun Pharmaceutical Industries Limited is an Indian multinational pharmaceutical company headquartered in Mumbai, Maharashtra, that manufactures and sells pharmaceutical formulations and active pharmaceutical ingredients in more than 100 countries across the globe.
Sun Pharmaceutical produces three final chemical products P_1, P_2 and P_3 requiring mixup of three raw material chemicals M_1, M_2 and M_3. The per unit requirement of each product for each material (in litres) is as follows:

$$A = \begin{matrix} & M_1 & M_2 & M_3 \\ P_1 & 2 & 3 & 1 \\ P_2 & 4 & 2 & 5 \\ P_3 & 2 & 4 & 2 \end{matrix}$$

(i) Find the total requirement of each material if the firm produces 100 litres of each product,
(ii) Find the per unit cost of production of each product if the per unit of materials M_1, M_2 and M_3 are ₹ 5, ₹ 10 and ₹ 5 respectively, and
(iii) Find the total cost of production if the firm produces 200 litres of each product.

37. Commodity prices are primarily determined by the forces of supply and demand in the market. For example, if the supply of oil increases, the price of one barrel decreases. Conversely, if demand for oil increases (which often happens during the summer), the price rises. Gasoline and natural gas fall into the energy commodities category.



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The price p (dollars) of each unit of a particular commodity is estimated to be changing at the rate

$$\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$$

where x (hundred) units is the consumer demand (the number of units purchased at that price). Suppose 400 units ($x = 4$) are demanded when the price is \$30 per unit.

(i) Find the demand function $p(x)$.

(ii) At what price will 300 units be demanded? At what price will no units be demanded?

(iii) How many units are demanded at a price of \$20 per unit?

38. Quality assurance (QA) testing is the process of ensuring that manufactured product is of the highest possible quality for customers. QA is simply the techniques used to prevent issues with product and to ensure great user experience for customers.

A manufactured component has its quality graded on its performance, appearance, and cost. Each of these three characteristics is graded as either pass or fail. There is a probability of 0.40 that a component passes on both appearance and cost. There is a probability of 0.35 that a component passes on both performance and appearance. There is a probability of 0.31 that a component passes on all three characteristics. There is a probability of 0.64 that a component passes on performance. There is a probability of 0.19 that a component fails on all three characteristics. There is a probability of 0.06 that a component passes on appearance but fails on both performance and cost.

(i) What is the probability that a component passes on cost but fails on both performance and appearance?

(ii) If a component passes on both appearance and cost, what is the probability that it passes on all three characteristics?

(iii) If a component passes on both performance and appearance, what is the probability that it passes on all three characteristics?

□□□□□□□

Sample Paper 16

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. Which of the following is correct for the function $f(x) = \sin 2x - 1$ at the point $x = 0$ and $x = \pi$

(a) Continuous at $x = 0,\ \pi$

(b) Discontinuous at $x = 0$ but continuous at $x = \pi$

(c) Continuous at $x = 0$ but discontinuous at $x = \pi$

(d) Discontinuous at $x = 0,\ \pi$

2. If $x = \dfrac{2\,at}{1+t^3}$ and $y = \dfrac{2\,at^2}{(1+t^3)^2}$, then $\dfrac{dy}{dx}$ is equal to

(a) ax (b) a^2x^2

(c) $\dfrac{x}{a}$ (d) $\dfrac{x}{2a}$

3. $\displaystyle\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x}\,dx$ is equal to

(a) $-\log(1+\sin^2 x) + C$ (b) $\log(1+\cos^2 x) + C$

(c) $-\log(1+\cos^2 x) + C$ (d) $\log(1+\tan^2 x) + C$

4. If $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ is a relation on the set $A = \{3,6,9,12\}$. Then, the relation is

(a) an equivalence relation (b) reflexive and symmetric

(c) reflexive and symmetric (d) only reflexive

5. $\displaystyle\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\,dx$ is equal to

(a) 0 (b) 2

(c) 4 (d) -2

6. The function $f(x) = x^2 - 2x$ is increasing in the interval

(a) $x \neq -1$ (b) $x \geq -1$

(c) $x \neq 1$ (d) $x \geq 1$

7. The minimum value of $f(x) = \sin x \cos x$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 0 (d) 5

8. A mapping $f : n \to N$, where N is the set of natural numbers is define as

$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases} \text{ for } n \in N. \text{ Then, } f \text{ is}$$

(a) Surjective but not injective (b) Injective but not surjective

(c) Bijective (d) neither injective nor surjective

9. The value of $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$ is

(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$

(c) $\frac{7}{25}$ (d) none of these

10. If $AB = A$ and $BA = B$, then B^2 is equal to

(a) B (b) A

(c) $-B$ (d) B^3

11. The area of enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(a) 12 sq units (b) 13 sq units

(c) $13\frac{1}{2}$ sq units (d) 14 sq units

12. The solution of $e^{dy/dx} = x + 1, y(0) = 3$, is

(a) $y = x\log x - x + 2$ (b) $y = (x+1)\log(x+1) - x + 3$

(c) $y = (x+1)\log(x+1) + x + 3$ (d) $y = x\log x + x + 3$

13. $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$ is

(a) $\log x = \log(x - y) + \frac{y}{x} + c$ (b) $\log x = 2\log(x - y) + \frac{y}{x} + c$

(c) $\log x = \log(x - y) + \frac{x}{y} + c$ (d) none of the above

14. The general solution of the differential equation $x(1+y^2)\,dx + y(1+x^2)\,dy = 0$ is

(a) $(1+x^2)(1+y^2) = 0$

(b) $(1+x^2)(1+y^2) = c$

(c) $(1+y^2) = c(1+x^2)$

(d) $(1+x^2) = c(1+y^2)$

15. If $\vec{a}$ and $\vec{b}$ are position vectors of A and B respectively, then the position vector of a point C in AB produced such that $\overrightarrow{AC} = 3\overrightarrow{AB}$, is

(a) $3\vec{a} - \vec{b}$

(b) $3\vec{b} - \vec{a}$

(c) $3\vec{a} - 2\vec{b}$

(d) $3\vec{b} - 2\vec{a}$

16. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = 2\hat{j} - \hat{k}$ and $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$, $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$, then $\dfrac{\vec{r}}{|\vec{r}|}$ is equal to

(a) $\dfrac{1}{\sqrt{11}}(\hat{i} + 3\hat{j} - \hat{k})$

(b) $\dfrac{1}{\sqrt{11}}(\hat{i} - 3\hat{j} + \hat{k})$

(c) $\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(d) none of these

17. The foot of the perpendicular from (0, 2, 3) to the line $\dfrac{x+3}{5} = \dfrac{y=1}{2} = \dfrac{z+4}{3}$ is

(a) $(-2, 3, 4)$

(b) $(2, -1, 3)$

(c) $(2, 3, -1)$

(d) $(3, 2, -1)$

18. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$, then $P\left(\dfrac{A'}{B}\right)$ is equal to

(a) $\dfrac{1}{3}$

(b) $\dfrac{1}{2}$

(c) $\dfrac{2}{3}$

(d) $\dfrac{3}{4}$

19. For any square matrix A with real number entries consider the following statements.
Assertion: $A + A'$ is a symmetric matrix.
Reason: $A - A'$ is a skew-symmetric matrix.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)P(B)$
Assertion: $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$

Reason: $P(A \cup B) = P(A) + P(B)$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not.

22. Evaluate $\int \cos^{-1}(\sin x)\, dx$.

OR

Write the anti-derivative of $\left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)$.

23. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude of 21 units.

OR

Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

24. What are the direction cosines of a line which makes equal angles with the coordinate axes?

25. Two groups are computing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product introduced way by the second group.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

27. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.

28. If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

OR

If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

29. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

OR

Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

30. Evaluate $\int \frac{2\cos x}{\sin^2 x} dx$.

31. Find the area of a parallelogram whose adjacent sides represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{i} + 2\hat{k}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also, find their point intersection.

33. Find $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$.

OR

Evaluate $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$.

34. Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$, given $y = 2$ when $x = \frac{\pi}{2}$.

35. Maximize $Z = 3x + 2y$ subject to $x + 2y \leq 0$, $3x + y \leq 15$, x, $y \geq 0$.

OR

Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, x, $y \geq 0$. Show that the minimum of Z occurs at more than two points.

Section - E

Case study based questions are compulsory.

36. Cross holding, also referred to as cross shareholding, describes a situation where one publicly-traded company holds a significant number of shares of another publicly-traded company. The shares owned of the second publicly-traded company are referred to as a cross-holding of the first company.

Two companies A and B are holding shares in each other. A is holding 20% shares of B and B is holding 10% shares. of A. The separately earned profits of the two companies are ₹ 98000 and ₹ 49000 respectively.

(i) Find total profit of each company using matrix notations.

(ii) Show that the total of the profits allocated to outside shareholders is equal to the total of separately earned profit.

37. Ravindra Manch was established in 1963 to commemorate the 100th birth anniversary of Ravindra Nath Tagore. Ravindra Manch is one of the myriad places in Jaipur that hold a historical significance. The auditorium was among the seventeen cultural centers that were envisioned by Pandit Jawaharlal Nehru and was thrown open to the public on Independence Day in the year 1963. Since then, the place has hosted a wide number of cultural shows and events. Some of the most renowned artists, dancers and actors have displayed their talent at this prestigious venue.

Last year, 300 people attended the Ravindra Manch Drama Club's winter play. The ticket price was ₹ 70. The advisor estimates that 20 fewer people would attend for each ₹ 10 increase in ticket price.

(i) What ticket price would give the most income for the Drama Club?

(ii) If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

38. Federal health officials have reported that the proportion of children (ages 19 to 35 months) who received a full series of inoculations against vaccine-preventable diseases, including diphtheria, tetanus, measles, and mumps, increased up until 2006, but has stalled since. The CDC reports that 14 states have achieved a vaccination coverage rate of at least 80% for the 4:3:1:3:3:1 series.26 The probability that a randomly selected toddler in Alabama has received a full set of inoculations is 0.792, for a toddler in Georgia, 0.839, and for a toddler in Utah, 0.711.27 Suppose a toddler from each state is randomly selected.

(i) Find the probability that all three toddlers have received these inoculations.

(ii) Find the probability that none of the three has received these inoculations.

□□□□□□□

Sample Paper 17

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. $\frac{d}{dx}(\sin x) =$

(a) $\cos x$ (b) $-\sin x$

(c) $-\cos x$ (d) $\tan x$

2. $f(x) = 2x^3 - 15x^2 + 36x + 4$ is

(a) increasing in $(-\infty, 2]$ (b) increasing in [2, 3]

(c) decreasing in $(3, \infty)$ (d) None of these

3. Maximum value of $f(x) = \sin x + \cos x$ is

(a) 1 (b) 2

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

4. The real number which most exceeds its cube is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) None of these

5. $\int_2^1 \frac{dx}{x} = ?$

(a) $\log\frac{2}{3}$

(b) $\log\frac{3}{2}$

6. If $A = \{1, 2, 3\}$, then how many equivalence relation can be defined on A containing (1, 2) :

(a) 2

(b) 3

(c) 8

(d) 6

7. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = ?$

(a) $\frac{\pi}{4}$

(b) $-\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $-\frac{\pi}{2}$

8. $A = \begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix}$, $2A + 3B = ?$

(a) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$

(b) $\begin{bmatrix} 27 & 36 \\ 25 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 27 & 36 \\ 25 & 15 \end{bmatrix}$

(d) $\begin{bmatrix} 27 & 36 \\ 35 & 10 \end{bmatrix}$

9. If $\lambda \in R$ and $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $\lambda\Delta =$

(a) $\begin{vmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{vmatrix}$

(b) $\begin{vmatrix} \lambda a & b \\ c & d \end{vmatrix}$

(c) $\begin{vmatrix} \lambda a & b \\ \lambda c & d \end{vmatrix}$

(d) None of these

10. $\frac{d}{dx}[\log x] = ?$

(a) $\frac{1}{x}$

(c) $-\frac{1}{x^2}$

(c) 1

(d) $\frac{1}{x^2}$

(c) $\log\frac{1}{2}$

(d) $\log\frac{x}{2}$

11. $\int \frac{xe^x}{(x+1)^2}dx =$

(a) $\frac{e^x}{(x+1)^2} + c$

(b) $\frac{-e^x}{x+1} + c$

(c) $\frac{e^x}{x+1} + c$

(d) $\frac{-e^x}{(x+1)^2} + c$

12. $\int 0.dx =$ ....

(a) k

(b) 0

(c) 1

(d) -1

13. Integrating factor (IF) of the differential equation

$$\frac{dy}{dx} - y\cos x = \sin x \cos x$$

(a) $e^{-\sin x}$

(b) $e^{\sin x}$

(c) $e^{-\cos x}$

(d) $e^{\cos x}$

14. The solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ is

(a) $x - y = k$

(b) $x^2 - y^2 = k$

(c) $x^3 - y^3 = k$

(d) $xy = k$

15. $\hat{i} \times (\hat{i} \times \hat{j}) + \hat{j} \times (\hat{j} \times \hat{k}) + \hat{k} \times (\hat{k} \times \hat{i}) =$

(a) $\hat{i} + \hat{j} + \hat{k}$

(b) 0

(c) 1

(d) $-(\hat{i} + \hat{j} + \hat{k})$

16. The direction ratios of a straight line are 1,3,5. Its direction cosines are

(a) $\frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}$

(b) $\frac{1}{9}, \frac{1}{3}, \frac{5}{9}$

(c) $\frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

(d) $\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

17. If the direction cosines of two straight lines are l_1, m_1, n_1 and l_2, m_2, n_2 then the cosine of the angle θ between them or $\cos\theta$ is

(a) $(l_1 + m_1 + n_1)(l_2 + m_2 + n_2)$

(b) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(c) $l_1 l_2 + m_1 m_2 + n_1 n_2$

(d) $\frac{l_1 + m_1 + n_1}{l_2 + m_2 + n_2}$

18. The optimal value of the objective function is attached at the point:

(a) given by intersection of inequations with axes only.

(b) given by intersection of inequations with x-axis only.

(c) given by corner points of the feasible region.

(d) none of the above.

19. **Assertion :** If A and B are two independent events $P(A\cup B)=1-P(A')P(B')$
Reason : $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ and $P(A\cap B)=P(A)P(B)$

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

20. **Assertion :** $f(x)$ is defined as $f(x)=\begin{cases}x^3-3, x\leq 2\\ x^2+1, x>2\end{cases}$ is continuous at $x=2$.
Reason : $f(2)=\lim_{x\to 2}f(x)$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $y=\sqrt{x+\sqrt{x+\sqrt{x+...\text{to}\,\infty}}}$ then $\frac{dy}{dx}$

OR

If $y=\tan(\sin^{-1}x)$ then find $\frac{dy}{dx}$

22. Show that the function $y=ae^{\tan^{-1}x}$ is a solution of the differential equation

$$(1+x^2)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}=0$$

OR

Show that the function $y=ax+\frac{b}{a}$ is a solution of the differential equation $y=x\frac{dy}{dx}+\frac{b}{\frac{dy}{dx}}$

23. Prove, by Vector method, that the angle inscribed in a semi-circle is a right angle.

24. Prove by direction numbers, that the point $(1,-1,3)$, $(2,-4,5)$ and $(5,-13,11)$ are in a straight line.

25. Odds are 8 : 5 against a man, who is 55 years old, living till he is 75 and 4 : 3 against his wife who is now 48, living till she is 68. Find the probability that the couple will be alive 20 years hence.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. If $f:R\to R$ is defined by $f(x)=x^2-3x+2$, find $f\{f(x)\}$.

$= x(x^3-6x^2+10x-3)$

27. Prove that: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

OR

Prove that $4(\cot^{-1}3+\operatorname{cosec}^{-1}\sqrt{5})=\pi$.

28. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, then find the value of $A^2+3A+2I$.

OR

Find the values of X and $Y: X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$.

29. If $y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$, show that $\frac{dy}{dx}-\sec x=0$.

30. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 cm long?

31. If $\vec{a}=\hat{i}-2\hat{j}-3\hat{k}$; $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}-2\hat{k}$ then find $\vec{a}\times(\vec{b}\times\vec{c})$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate $\int \frac{5x+11}{\sqrt{9x^2+25}}dx$

OR

Prove that : $\int_0^{\pi/2} \log(\tan\theta + \cot\theta)\, d\theta = \pi \log 2$

33. Solve the differential equation: $(x-y)\,dy - (x+y)\,dx = 0$

OR

Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$.

34. Solve the following L.P.P. graphically:
Minimise and Maximise

$$Z = 3x + 5y$$

Subject to the constraints:

$$3x - 4y + 12 \geq 0$$
$$2x - y + 2 \geq 0$$
$$2x + 3y - 12 \geq 0$$
$$x \leq 4$$
$$y \geq 2$$
$$x \geq 0$$

35. A random variable has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) k

(ii) $P(X<3)$

(iii) $P(X>6)$

(iv) $P(0<x<3)$

Section - E

Case study based questions are compulsory.

36. Pastry is a dough of flour, water and shortening that may be savoury or sweetened. Sweetened pastries are often described as bakers' confectionery. The word "pastries" suggests many kinds of baked products made from ingredients such as flour, sugar, milk, butter, shortening, baking powder, and eggs.

The Sunrise Bakery Pvt Ltd produces three basic pastry mixes A, B and C. In the past the mix of ingredients has shown in the following matrix:

$$\text{Type}\ \begin{matrix} A \\ B \\ C \end{matrix}\overset{\begin{matrix}\text{Flour} & \text{Fat} & \text{Sugar}\end{matrix}}{\begin{bmatrix} 5 & 1 & 1 \\ 6.5 & 2.5 & 0.5 \\ 4.5 & 3 & 2 \end{bmatrix}} \quad \text{(All quantities in kg)}$$

Due to changes in the consumer's tastes it has been decided to change the mixes using the following amendment matrix:

$$\text{Type}\ \begin{matrix} A \\ B \\ C \end{matrix}\overset{\begin{matrix}\text{Flour} & \text{Fat} & \text{Sugar}\end{matrix}}{\begin{bmatrix} 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \end{bmatrix}}$$

Using matrix algebra you are required to calculate:
(i) the matrix for the new mix:
(ii) the production requirement to meet an order for 50 units of type A, 30 units of type B and 20 units of type C of the new mix;
(iii) the amount of each type that must be made to totally use up 370 kg of flour, 170 kg of fat and 80 kg of sugar that are at present in the stores.

37. steel can, tin can, tin, steel packaging, or can is a container for the distribution or storage of goods, made of thin metal. Many cans require opening by cutting the "end" open; others have removable covers. They can store a broad variety of contents: food, beverages, oil, chemicals, etc.

A tin can manufacturer a cylindrical tin can for a company making sanitizer and disinfector. The tin can is made to hold 3 litres of sanitizer or disinfector.

Based on the above information, answer the following questions.

(i) If r be the radius and h be the height of the cylindrical tin can, find the surface area expressed as a function of r.

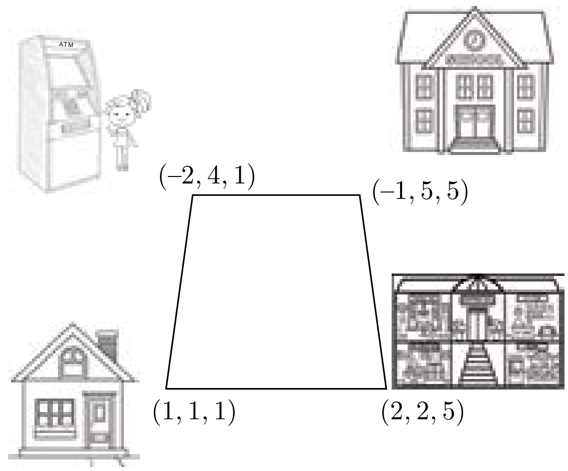
(ii) Find the radius that will minimize the cost of the material to manufacture the tin can.

(iii) Find the height that will minimize the cost of the material to manufacture the tin can.

OR

(iv) If the cost of the material used to manufacture the tin can is ₹100/m^2 find the minimum cost. $\sqrt[3]{\frac{1500}{\pi}} \approx 7.8$

38. Lavanya starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to ATM, from there to her daughter's school and then reaches the mall. In the diagram, using co-ordinate geometry the location of each place is given.



Based on the above information, answer the following questions.

(i) What is the distance between House and ATM ?

(ii) What is the distance between ATM and school?

(iii) What is the total distance travelled by Lavanya?

OR

(iv) What is the extra distance travelled by Lavanya in reaching the shopping mall?

Sample Paper 18

Mathematics (Code-041)

Class XII Session 2022-23

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. $\frac{d}{dx}\left[\lim_{x \to a}\frac{x^5 - a^5}{x - a}\right] =$

(a) $5a^4$ (b) $5x^4$

(c) 1 (d) 0

2. The radius of a circle is increasing at the rate of 0.4 cm/s. The rate of increase of its circumference is

(a) 0.4π cm/s (b) 0.8π cm/s

(c) 0.8 cm/s (d) None of these

3. A ball thrown vertically upwards according to the formula $s = 13.8t - 4.9t^2$, where s is in metres and t is in seconds. Then its velocity at $t = 1$ sec is

(a) 6m/ sec (b) 4 m/ sec

(c) 2 m/ sec (d) 8 m/ sec

4. The maximum value of $y = 2x^3 - 21x^2 + 36x - 20$ is

(a) -128 (b) -126

(c) -120 (d) None of these

5. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x\, dx = ?$

(a) -1
(b) 0
(c) 1
(d) $\frac{\pi}{2}$

6. What type of a relation is "Less than" in the set of real numbers?

(a) only symmetric
(b) only transitive
(c) only reflexive
(d) equivalence relation

7. $\tan^{-1} x + \cot^{-1} x = ?$

(a) 0
(b) 1
(c) $\frac{\pi}{2}$
(d) $-\frac{\pi}{2}$

8. Which of the following is the unit matrix of order 3×3?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} \sin 20^\circ & -\cos 20^\circ \\ \sin 70^\circ & \cos 70^\circ \end{bmatrix} = ?$

(a) 1
(b) -1
(c) 0
(d) 2

10. $\frac{d}{dx}[\tan x] = ?$

(a) $\sec^2 x$
(b) $\sec x$
(c) $\cot x$
(d) $-\sec^2 x$

11. $\int x^2 \cdot e^{x^3}\, dx =$

(a) $e^{x^3} + c$
(b) $\frac{1}{3} e^{x^3} + c$
(c) $e^{x^2} + c$
(d) $\frac{1}{3} e^{x^2} + c$

12. $\int x^5\,dx =$ ....

(a) $\frac{x^6}{6}+k$ (b) $\frac{x^5}{5}+k$

(c) $\frac{x^7}{7}+k$ (d) $\frac{x^8}{8}+k$

13. Which of the following is a homogeneous differential equation?

(a) $x^2y dx-(x^2+y^2)dy=0$ (b) $(xy)dx-(x^4+y^4)dy=0$

(c) $(2x+y-3)dy-(x+2y-3)dx=0$ (d) $(x-y)dy=(x^2+y+1)dx$

14. The degree of the equation $\left(\frac{d^2y}{dx^2}\right)^2-x\left(\frac{dy}{dx}\right)^3=y^3$ is

(a) 0 (b) 1

(c) 2 (d) 3

15. The direction cosines of the vector $3\hat{i}-4\hat{j}+12\hat{k}$ is

(a) $\frac{3}{13},\frac{4}{13},\frac{12}{13}$ (b) $\frac{3}{13},\frac{-4}{13},\frac{12}{13}$

(c) $\frac{3}{\sqrt{13}},\frac{4}{\sqrt{13}},\frac{12}{\sqrt{13}}$ (d) $\frac{3}{\sqrt{13}},\frac{-4}{\sqrt{13}},\frac{12}{\sqrt{13}}$

16. If l, m, n are the direction cosines of a straight line then

(a) $l^2+m^2-n^2=1$ (b) $l^2-m^2+n^2=1$

(c) $l^2-m^2-n^2=1$ (d) $l^2+m^2+n^2=1$

17. The direction ratios of the line joining the points (x_1,y_1,z_1) and (x_2,y_2,z_2) are

(a) $x_1+x_2,\ y_1+y_2,\ z_1+z_2$

(b) $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

(c) $\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}$

(d) $(x_2-x_1),\ (y_2-y_1),\ (z_2-z_1)$

18. Of all the points of the feasible region, for maximum or minimum of objective functions, the point lies:

(a) inside the feasible region

(b) at the boundary line of the feasible region

(c) vertex point of the boundary of the feasible region

(d) none of the above

19. A and B are two events
Statement-I : If $P(\overline{A}) = 0.7$, $P(\overline{B}) = 0.5$ and $P(A \cup B) = 0.6$ then $P(A \cap B) = 0.2$
Reason : $P(A \cup B) + P(A \cap B) + P(\overline{A}) + P(\overline{B}) = 2.5$

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

20. **Assertion : :** If $y = x^3 \cos x$, then $\frac{dy}{dx} = x^3 \sin x + 3x^2 \cos x$

Reason : $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find $\frac{dy}{dx}$ if $y = \cos\sqrt{\sin x}$

OR

Find $\frac{dy}{dx}$, when $x = y \log(xy)$

22. Solve : $\frac{dy}{dx} + 2y \tan x = \sin x$.

OR

Show that the function $1 + 8y^2 \tan x = ay^2$ is a solution of differential equation

$$\cos^2 x \frac{dy}{dx} = 4y^3$$

23. Prove that – $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a}\,\vec{b}|^2$

24. Show that the line joining the points (4, 7, 8), (2, 3, 4) is parallel to the line joining the points (2, 4, 10), (–2, –4, 2).

25. If A and B are two independent events then prove that : $P(A\cup B) = 1 - P(A')\cdot P(B')$

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. In the set Q of all rational numbers, a binary operation $o : Q\times Q \to Q$ is defined by $o(x,y) = x\,o\,y = x + y - xy$ then show that o is commutative.

27. Find the value of $\cot^{-1}\left(\tan\frac{\pi}{7}\right)$?

OR

Prove that $4(\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5}) = \pi$.

28. Find the value of x, such that $\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}\begin{bmatrix}x & -5 & -1\end{bmatrix} = 0$

OR

If $A = \begin{bmatrix}2 & 5\\ 3 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 5\\ 6 & 2\end{bmatrix}$ then find $(A+B)$ and $(A-B)$.

29. If $f(x) = x\sin\frac{1}{x}$, when $x \neq 0$; and, $f(x) = 0$, when $x = 0$, then test the continuity of $f(x)$ at $x = 0$.

30. The radius of a circle is increasing uniformly at the rate of 3 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

31. Find the value of p, if $(2\hat{i} + 6\hat{j} + 27\hat{k})\times(\hat{i} + 3\hat{j} + p\hat{k}) = 0$

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate $\int \frac{-7x+2}{\sqrt{16x^2-9}}dx$

OR

Prove that $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2}\log 2$

33. Solve : $\frac{dy}{dx} - \frac{2y}{x} = y^4$

OR

Solve $y^2 dx + (x^2 + xy)\, dy = 0$

34. Solve the following L.P. problem graphically:

Maximise $Z = x + y$

Subject to $x - y \leq -1$

$-x + y \leq 0$

$x, y \geq 0$

35. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs in drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Section - E

Case study based questions are compulsory.

36. Rice is a nutritional staple food which provides instant energy as its most important component is carbohydrate (starch). On the other hand, rice is poor in nitrogenous substances with average composition of these substances being only 8 per cent and fat content or lipids only negligible, i.e., 1per cent and due to this reason it is considered as a complete food for eating. Rice flour is rich in starch and is used for making various food materials.

Two farmers Ramkishan and Gurcharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in ₹) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.
September Sales (in ₹)

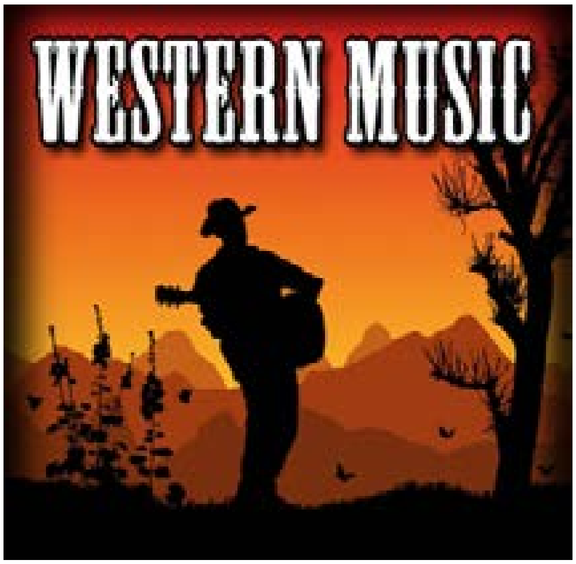
$$\begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ A = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array} \end{array}$$

October Sales (in ₹)

$$\begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ B = \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array} \end{array}$$

(i) Find the combined sales in September and October for each farmer in each variety.
(ii) Find the decrease in sales from September to October.
(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

37. Western music is a form of country music composed by and about the people who settled and worked throughout the Western United States and Western Canada. Western music celebrates the lifestyle of the cowboy on the open ranges, Rocky Mountains, and prairies of Western North America.



Western music is organised every year in the stadium that can hold 36000 spectators. With ticket price of ₹10, the average attendance has been 24000. Some financial expert estimated that price of a ticket should be determined by the function $p(x) = 15 - \frac{x}{3000}$, where x is the number of ticket sold.
Bases on the above information, answer of the following questions.
(i) Find the expression for total revenue R as a function of x.

(ii) Find the value of x for which revenue is maximum.

(iii) When the revenue is maximum, what will be the price of the ticket?

OR

(iv) How many spectators should be present to maximum the revenue?

38. If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

Based on the above information, answer the following questions.

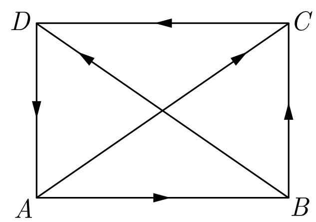
(i) If $\hat{p}$, $\hat{q}$, $\hat{r}$ are the vectors represented by the side of a triangle taken in order, then find $\vec{q} + \vec{r}$.

(ii) If $ABCD$ is a parallelogram and AC and BD are its diagonals, then find $\overrightarrow{AC} + \overrightarrow{BD}$.

(iii) If $ABCD$ is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find $\overrightarrow{AC} - \overrightarrow{BD}$.

OR

(iv) If ABCD is a quadrilateral, whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then find $\overrightarrow{BA} + \overrightarrow{CD}$.



Sample Paper 19

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $y = \log\cos x^2$, then $\frac{dy}{dx}$ at $x = \sqrt{\pi}$ has the value

(a) 1 (b) $\frac{\pi}{4}$

(c) 0 (d) $\sqrt{\pi}$

2. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

(a) 1 (b) e

(c) $e^{1/e}$ (d) None of these

3. At time t distance of a particle moving in a straight line is given by $s = 4t^2 + 2t$. The acceleration when $t = \frac{1}{2}$, is

(a) 4 (b) 6

(c) 8 (d) 10

4. If the distance travelled by a particle in time t is $s = 180t - 16t^2$, then the rate of change in velocity is

(a) 48 unit (b) -32 unit

(c) $-16t$ unit (d) none of these

5. $\int_0^1 \frac{dx}{1+x^2} = ?$

(a) $\frac{-\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{-\pi}{2}$

6. $f : A \to B$ will be an onto function if-

(a) $f(A) \subset B$

(b) $f(A) = B$

(c) $f(A) \supset B$

(d) $f(A) \neq B$

7. If $-1 < x < 1$, then $2\tan^{-1}x = ?$

(a) $\sin^{-1}\frac{2x}{1+x^2}$

(b) $\sin^{-1}\frac{2x}{1-x^2}$

(c) $\sin^{-1}\frac{1-x^2}{1+x^2}$

(d) $\sin^{-1}\frac{1+x^2}{1-x^2}$

8. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then-

(a) $(x = -2, y = 8)$

(b) $(x = 2, y = -8)$

(c) $(x = 3, y = -6)$

(d) $(x = -3, y = 6)$

9. The value of x when $\begin{bmatrix} x & 15 \\ 4 & 4 \end{bmatrix} = 0$ is-

(a) 15

(b) –15

(c) 4

(d) $4x$

10. $\frac{d}{dx}[\sin^2 x] = ?$

(a) $2\sin x\cos x$

(b) $2\sin x$

(c) $\cos^2 x$

(d) $\sin x\cos x$

11. $\int\sqrt{1+\cos 2x}\, dx =$

(a) $\sqrt{2}\cos x + c$

(b) $\sqrt{2}\sin x + c$

(c) $-\cos x - \sin x + c$

(d) $\sqrt{2}\sin\frac{x}{2} + c$

12. Solution of $\lim_{n \to \infty}\left[\frac{e^{1/n}+e^{2/n}+e^{3/n}+...+e^{r/n}}{n}\right]$

(a) $1-e$

(b) $e-1$

(c) e

(d) 1

13. Integrating factor of the differential equation $\frac{dy}{dx}+y\sec x=\tan x$ is-

(a) $\sec x+\tan x$

(b) $\sec x-\tan x$

(c) $\sec x$

(d) $\tan x\sec x$

14. The integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$ is

(a) $e^{\int Pdy}$

(b) $e^{\int Qdx}$

(c) $e^{\int Qdy}$

(d) $e^{\int Pdx}$

15. If $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-\hat{k}$, then the value of $(\vec{a}+3\vec{b}).(2\vec{a}-\vec{b})$ is-

(a) 15

(b) 18

(c) -18

(d) -15

16. Let l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two straight lines. Both the lines are perpendicular to each other, if

(a) $l_1l_2+m_1m_2+n_1n_2=0$

(b) $l_1l_2+m_1m_2+n_1n_2=1$

(c) $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$

(d) $\frac{l_1}{l_2}+\frac{m_1}{m_2}+\frac{n_1}{n_2}=0$

17. The direction cosines of the $y-$axis are

(a) (0, 0, 0)

(b) (1, 0, 0)

(c) (0, 1, 0)

(d) (0, 0, 1)

18. Feasible region is the set of points which satisfy:

(a) the objective function

(b) some of the given constraints

(c) all the given constraints

(d) none of these

19. **Assertion :** If $P(E_1) = 0.3$ and $P(\overline{E_2}) = 0.6$ then $P(E_1 \cup E_2) = 0.7$

Reason : $P(\overline{E_2}) = 1 - P(E_2)$

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

20. **Assertion :** If $y = \sin x^3$, then $\dfrac{dy}{dx} = \cos x^3 \cdot 3x^2$.

Reason: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find $\dfrac{dy}{dx}$ if $y = \cos^{-1}\dfrac{1-x^2}{1+x^2}$

OR

Find $\dfrac{dy}{dx}$ if $y = \sqrt{\sin x}$

22. Solve $-\dfrac{dy}{dx} = e^{x-y}$

OR

Write down the order and the degree of the equation :

$$8x^2\frac{d^2y}{dx^2} - 7\left(\frac{dy}{dx}\right)^2 + 9 = 0$$

23. If $\vec{a} = (2, 3, -5)$ and $\vec{b} = (2, 2, 2)$, then prove that $\vec{a}$ and $\vec{b}$ are mutually perpendicular.

24. The direction ratios of a straight line are 1,3,5. Find its direction cosines.

25. Two dice are thrown. Find the probability that the numbers appearing have a sum 8 if it is known that the second die always exhibits 4.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Is the function $f: R \to R$ onto function (where $f(x) = 2x$) Give reasons.

27. Prove that : $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$

OR

Find the value of $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$.

28. If $A = \begin{bmatrix} 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 10 & 9 \\ 8 & 7 & 6 \end{bmatrix}$, then find $A + B =$

OR

If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then prove that $f(x+y) = f(x).f(y)$

29. If $y = \tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right]$, prove that $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$.

30. How fast is the volume of a ball changing with respect to its radius when radius is 3 m?

31. Prove, by Vector method, that the angle inscribed in a semi-circle is a right angle.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate $\int \frac{3-5x}{2x^2+3x-2}\,dx$

OR

Prove that : $\int_0^{\pi/2} \log\sin x\,dx = -\frac{\pi}{2}\log 2$

33. Solve : $\sqrt{a+x}\frac{dy}{dx}+x=0$

OR

Solve : $\frac{dy}{dx}-\frac{xy}{1-x^2}=\frac{1}{1-x^2}$

34. Solve the following L.P. Problem graphically.

Maximise $Z=-x+2y$

Subject to the constraints:

$$x \geq 3$$
$$x+y \geq 5$$
$$x+2y \geq 6$$
$$y \geq 0$$

35. Find the probability distribution of the number of succes in two tosses of a die, where a succes is defined as

(i) number greater than 4

(ii) six appears on at least one die.

Section - E

Case study based questions are compulsory.

36. A car carrier trailer, also known as a car-carrying trailer, car hauler, or auto transport trailer, is a type of trailer or semi-trailer designed to efficiently transport passenger vehicles via truck. Commercial-size car carrying trailers are commonly used to ship new cars from the manufacturer to auto dealerships. Modern car carrier trailers can be open or enclosed. Most commercial trailers have built-in ramps for loading and off-loading cars, as well as power hydraulics to raise and lower ramps for stand-alone accessibility.

A transport company uses three types of trucks T_1, T_2 and T_3 to transport three types of vehicles

V_1, V_2 and V_3. The capacity of each truck in terms of three types of vehicles is given below:

	V_1	V_2	V_3
T_1	1	3	2
T_2	2	2	3
T_3	3	2	2

Using matrix method find:

(i) The number of trucks of each type required to transport 85, 105 and 110 vehicles of V_1, V_2 and V_3 types respectively.

(ii) Find the number of vehicles of each type which can be transported if company has 10, 20 and 30 trucks of each type respectively.

37. Due to the growing need to reduce emissions across the world, most countries are replacing existing fuel-based transportation with electric mobility. India, still in the very early stages of adopting EV's can be a huge market for E-bike rental companies. As most people are now aware of traffic congestion and pollution, India is looking towards a cleaner mode of transportation to tackle the situation.

An owner of an electric bike rental company have determined if they charge customers ₹x per day to rent a bike, where $50 \leq x \leq 200$then number of bikes (n), they rent per day can be shown by linear function $n(x) = 2000 - 10x$. If they charge ₹50 per day or less, they will rent all their bikes. If they charges ₹200 or more per day they will not rent any bike.

Based on the above information, answer the following questions.

(i) Find the expression for total revenue R as a function of x.

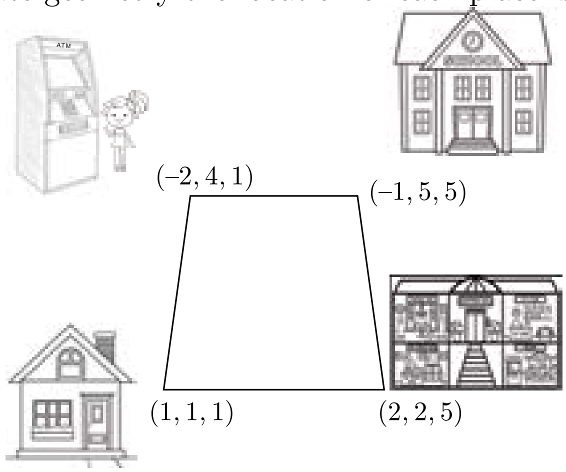
(ii) Find the value of x at maximum revenue.

(iii) What is the revenue collected by the company at $x = 260$? Find the number of bikes rented per day, if $x = 105$.

OR

(iv) Find the maximum revenue, collected by company.

38. Lavanya starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to ATM, from there to her daughter's school and then reaches the mall. In the diagram, using co-ordinate geometry the location of each place is given.



Based on the above information, answer the following questions.

(i) What is the distance between House and ATM ?

(ii) What is the distance between ATM and school?

(iii) What is the total distance travelled by Lavanya?

OR

(iv) What is the extra distance travelled by Lavanya in reaching the shopping mall?

□□□□□□□

Sample Paper 20

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. $\dfrac{d(2^x)}{d(3^x)} =$

(a) $\left(\dfrac{2}{3}\right)^x$

(b) $\dfrac{2^{x-1}}{3^{x-1}}$

(c) $\left(\dfrac{2}{3}\right)^x \log_3 2$

(d) $\left(\dfrac{2}{3}\right)^x \log_2 3$

2. The maximum value of $f(x) = \dfrac{\log x}{x}$ is.

(a) 1

(b) $\dfrac{2}{e}$

(c) e

(d) $\dfrac{1}{e}$

3. If a particle is moving such that the velocity acquired is proportional to the square root acceleration is :

(a) a constant

(b) $\propto S^2$

(c) $\propto \dfrac{1}{S^2}$

(d) $\propto S$

4. If a particle moves in a straight line so that $s = \frac{1}{2}vt$, then acceleration is

(a) a constant

(b) proportional to t

(c) proportional to v

(d) proportional to s

5. $\int \text{cosec}^2 x\, dx = ?$

(a) $\tan x + c$

(b) $-\cot x + c$

(c) $2\,\text{cosec}\, x + c$

(d) $-2\,\text{cosec}\, x + c$

6. In the set of all straight lines in a plane, the relation R "to be Perpendicular" is-

(a) Reflexive and transitive

(b) Symmetric and transitive

(c) Symmetric

(d) None of these

7. $\sin^{-1}\frac{1}{x} = ?$

(a) $\sec^{-1} x$

(b) $\text{cosec}^{-1} x$

(c) $\tan^{-1} x$

(d) $\sin x$

8. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, then which one of the following is equal to A'?

(a) $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 4 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 1 & 4 \end{bmatrix}$

9. Evaluate $\begin{bmatrix} 1 & -1 \\ y & x \end{bmatrix}$

(a) $x + y$

(b) $x - y$

(c) $-y - x$

(d) $1 - x$

10. $\frac{d}{dx}[\sin^{-1} x + \cos^{-1} x] = ?$

(a) 0

(b) $\frac{1}{\sqrt{1 - x^2}}$

(c) $-\frac{1}{\sqrt{1 - x^2}}$

(d) $\frac{1}{2}\sqrt{1 - x^2}$

11. $\int \frac{dx}{1 + x^2} = ?$

(a) $\tan x + c$

(b) $\tan^2 x + c$

(c) $\cot x + c$

(d) $-\cot^{-1} x + c$

12. $\int \frac{dx}{x+\sqrt{x}}$

(a) $\log x + \log(1+\sqrt{x}) + C$

(b) $2\log(1+\sqrt{x}) + C$

(c) $\log(1+\sqrt{x}) + C$

(d) $\log\sqrt{x} + C$

13. The order of the differential equation $\frac{d^2y}{dx^2} + x^3\left(\frac{dy}{dx}\right)^3 = x^4$ is-

(a) 1

(b) 2

(c) 4

(d) 3

14. The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^x + e^{-y} + k = 0$

(b) $e^{2x} = ke^y$

(c) $e^x = ke^{2y}$

(d) $e^x = ke^y$

15. The modulus of the vector $19\hat{i} + 5\hat{j} - 6\hat{k}$ is.

(a) $\sqrt{322}$

(b) $\sqrt{420}$

(c) $\sqrt{421}$

(d) $\sqrt{422}$

16. The distance of the point $(3,4,5)$ from x-axis is-

(a) 3

(b) 5

(c) $\sqrt{41}$

(d) None of these

17. The coordinates of the midpoint of the line segment joining the points (2, 3, 4) and (8, $-$ 3,8) are

(a) (10, 0, 12)

(b) (5, 6, 0)

(c) (6, 5, 0)

(d) (5, 0, 6)

18. Which one of the following statements is correct

(a) Every L.P.P admits an optimal solution

(b) A L.P.P admits unique optimal solution

(c) The optimal value occurs at a corner point of the feasible region

(d) If a L.P.P admits two optimal solutions, then it has infinite optimal solutions

19. **Assertion :** $f(x) = x^3$ is continuous at $x = 2$.

Reason: $f(x)$ is continuous at $x = a$ if

$$\lim_{x \to a^+} \int f(x) = \lim_{x \to a^-} f(x) = f(a)$$

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

20. **Assertion :** If A and B be two events corresponding to sample space S such that $P(A) = 0.2$ and $P(B) = 0.8$, then $A \cup B$ is a sure event.

Reason- If A and B are mutually exclusive events, then $P(A) + P(B) = 1$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find $\frac{dy}{dx}$ if $y = \tan(x + y)$

OR

Find $\frac{dy}{dx}$ if $y = \sin^3 x \cos^5 x$

22. Solve – $\frac{dy}{dx} + y \cot x = 2 \cos x$

OR

Solve – $x dy + y dx = 0$

23. If $\vec{a} = 2\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{b} = -7\hat{i} + 6\hat{j} + 8\hat{k}$ find $\vec{a} \times \vec{b}$

24. Find distance between (4, 3, 7) and $(1, -1, -5)$?

25. If $P(A) = \frac{2}{5}, P(B) = \frac{3}{5}, P(A \cup B) = \frac{3}{4}$, then find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Examine whether the function $f: R \to R$ is one-one (injective) if $f(x) = x^3,\ x \in R$.

27. Prove that $\tan^{-1}\sqrt{x} - \frac{1}{z}\cos^{-1}\frac{1-x}{1+x}$

OR

Prove that $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) = 15$

28. If $A = \begin{bmatrix} 5 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 & 1 \\ 6 & 8 & 4 \end{bmatrix}$, find AB.

OR

Find the value of x and y if-

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

29. If $y = \sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$, find $\frac{dy}{dx}$.

30. Find the rate of change of the area of a circle with respect to its radius r when (i) $r = 3$ cm and (ii) $r = 5$ cm.

31. Prove that – $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}\,\vec{b}|^2$

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate $\int \frac{5x-2}{1+2x+3x^2}dx$

OR

Evaluate: $\int_0^{\frac{\pi}{4}} (\tan x - x)\tan^2 x dx$.

33. Solve $(x^2 - y^2)\frac{dy}{dx} = 2xy$

OR

Solve $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

34. Solve graphically the following L.P. problem:

Maximise $Z = x + y$

Subject to the linear constraints

$$y - 2x - 1 \le 0$$
$$x \le 2$$
$$x + y \le 3$$

and $x,\, y \ge 0$

35. Find the probability distribution of

(i) number of heads in two tosses of a coin.

(ii) number of tails in the simultaneous tosses of three coins.

(iii) number of heads in four tosses of a coin.

Section - E

Case study based questions are compulsory.

36. The D.A.V. College Managing Committee, familiarly known as DAVCMC, is a non-governmental educational organisation in India and overseas with over 900 schools. 75 colleges and a university. It is based on the ideals of Maharishi Dayanand Saraswati. Full Form of DAV is Dayanand Anglo Vedic.

In a certain city there are 50 colleges and 400 schools. Each school and college has 18 peons, 5 clerks and 1 cashier. Each college in addition has 1 section officer and one librarian. The monthly salary of each of them is as follows:

Peon-₹ 3000, Clerk-₹ 5000, Cashier-₹ 6000, Section Officer-₹ 7000 and Librarian-₹ 9000

Using matrix notation, find

(a) total number of posts of each kind in schools and colleges taken together.

(b) the total monthly salary bill of all the schools and colleges taken together.

37. Minimum Support Price (MSP) is a form of market intervention by the Government of India to insure agricultural producers against any sharp fall in farm prices. The minimum support prices are announced by the Government of India at the beginning of the sowing season for certain crops on the basis of the recommendations of the Commission for Agricultural Costs and Prices (CACP). MSP is price fixed by Government of India to protect the producer - farmers - against excessive fall in price during bumper production years. The minimum support prices are a guarantee price for their produce from the Government. The major objectives are to support the farmers from distress sales and to procure food grains for public distribution. In case the market price for the commodity falls below the announced minimum price due to bumper production and glut in the market, government agencies purchase the entire quantity offered by the farmers at the announced minimum price.

The Government declare that farmers can get ₹300 per quintal for onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Ramawatar has 80 quintal of onions in the field on 1st July and he estimated that crop is increasing at the rate of 1 quintal per day.

Based on the above information, answer the following questions.

(i) If x is the number of days after 1st July then find the price and quantity of onion in terms of x.

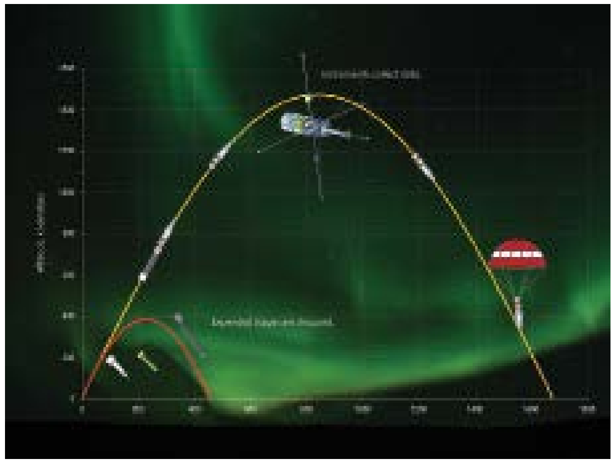
(ii) Find the expression for the revenue as a function of x.

(iii) Find the number of days after 1st July, when Ramawatar attain maximum revenue.

OR

(iv) On which day should Ramawatar harvest the onions to maximum his revenue? What is this maximum revenue?

38. Rocket motion is based on Newton's third law, which states that "for every action there is an equal and opposite reaction". Hot gases are exhausted through a nozzle of the rocket and produce the action force. The reaction force acting in the opposite direction is called the thrust force.



The equation of motion of a rocket are : $x = 2t$, $y = -4t$, $z = 4t$, where the time t is given in seconds, and the distance measured is in kilometres.

Based on the above information, answer the following questions.

(i) What is the path of the rocket? Which of the following points lie on the path of the rocket?

(ii) At what distance will the rocket be from the starting point (0, 0, 0) in seconds?

(iii) At certain instant of time, if the rocket is above sea level, where equation of surface of sea is given by $3x - y + 4z = 2$ and position of rocket at that instant of time is (1, –2, 2), then find the image of position of rocket in the sea.

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